**CHAPTER ONE**

INTRODUCTION AND PRELIMINARIES

* 1. **Introduction**

A magic square is a square grid of cells such that every row and column add up to the same number, called magic sum. Magic Square has been investigated for over 4120 years (Anup *et al.,* 2012), but throughout ages, it was lost in mathematics. The loss is attributed to astronomers. The consequence of the loss is that algebraic development of magic squares is not rich.

The oldest magic square recorded is the Lo Shu Magic Square. De La Loub, a French diplomat, devised a method of constructing the Lo Shu Magic Square in his book: “A New Historical Relation of the Kingdom of Siam”. By the Loub Magic Square, understand the magic square constructed with the Loub Method. The work is exclusively on the square andgiven our search no literature of this title typeexist much less its algebraic structural contents. Be that as it may; in this dissertation, the investigation of semigroups, groups, semirings and fields over the aforementioned square is pioneered. It is showcased that the Loub Magic Squares form the algebraic structures if equipped with some chosen binary and unary operationsminding the underlying multiset of entries in its cells.

Among the algebraic structures over the aforementioned squares are: The Tetrabonacci Group, Symmetry Group of Odd Lengths, Transformation Semigroup of Odd Lengths and Rhotrix Subelement Group. Diversification and incorporation inculcate these themes; and what is found across the board is that generalizations of these realms are not sidetracked. However, the following Loub Magic Square Algebraic Structures: Loub Magic Squares over Multiset of Natural Number Semigroup, Loub Magic Squares over Multiset of Integer Number Group, Minimum Zero-Centre Pandiagonal Composite Type II(a)Loub Magic Squares over Multiset of Natural Number Semiring, Loub Magic Squares over Multiset of Rational Numberof the form Infinite Field, Composite Loub Magic Squares Infinite Abelian Group, Type I Composite of Type I Loub Magic Squares Infinite Additive Abelian Group and of Type II Infinite Field, Minimal Bull Eye Composite Loub Magic Square Infinite Additive Abelian Groups, and Null Magic Sum and Null Magic Product Bull Eye Composite Loub Magic Ring are constructed, see also (Babayo, 2015).

Needless to highlight,a refined procedure and a new generalization of the Loubis introduced as well as the introduction and the proof of some important results. Also, the general centre piece and magic sum formula(s) are derived, and it is proved that the two formula(s) form respective infinite additive abelian groups. The investigation of Loub Magic Square Eigen Abelian Group is pioneered. This is meaningful for the principal value of the eigen values corresponds to the magic sum, see (Daryl, 1993).

The Rhotrix Subelements of the square are the rhomboidal array of number that are sitting in the magic squares. The matrix subelements have analogous meaning and are considered a babyish tautology, but the rhotrix subelements and the matrix subelements together form the method: Rhotrix-Matrix Construction of the Loub Magic Squares. This implies that one of the newest realms of mathematics, the Rhotrix Theory has now gotten another nonverbatim virtuoso matrix theory application.A juxtaposition alike of the construction of magic squares is coiled with an infix adjoinment of zeros to be referred to as the rhotrix-matrix construction for there exists an inter switch of matrix and rhotrix transformation systems which forces to provide the magic squares. It is showcasedvia concrete examples that the Rhotrix Subelements of the aforementioned squares form an infinite additive abelian group.

The notion of subelements and the study of the properties and inter properties of the square subelements of the Loub Magic Squares are laconically introduced. The most paramount importance of this lies in the fact that we got a method of construction of basic squares that are mostly pancolumn and that have no already known method of their construction earlier.

However, along with caution of the Cayley Theorem, the minimum number of the Loub Squares that will completely cover the structure of all the elements of the Transformation Semigroup of odd length maps is found. If the Loub Magic Square subsets are able to construct both the permutation subsets of the collection of maps from a set to itself and both the maps form respective Symmetry Group (Vasistha and Vasistha, 2006) and (Joseph, 2005) and Transformation Semigroup(Howie, 2003) with respect to the composition of maps and the set of the Loub Magic Squares equipped or enclosed with matrix binary operation of addition is an abelian group, then the Cayley Theorem for both the group and the semigroup still stand while all the job joyously jugs for the subsets of the squares so considered are not groups or semigroups by themselves.

The order of is and the order of , and In this dissertation, it is showcased that the number of the Loub Magic Square mappings will generate number of all maps to be equipped with composition to form the transformation semigroup. To economize space, we present a concrete example of the since Loub Magic Square is a triviality and , the oddest (even) prime, does not exist. The n has to always be odd matching the exact definition of the Loub Magic Square.We also highlighted consortium of miscellany effects of rotations and/or reflections [see also Gan *et al*., (2012)] and/or enumerations of the Loub Magic Squares to figure out the consortium of the Composites. We refer to (Lee, 1986) for the miscellany effects of rotations and/or reflections of the magic square and referto (Ahmed, 2004) for the definition of Composite Magic Square.

* 1. **Statement of the Problem**

The idea of algebraic structural properties of Magic Squares is conceived from the published research papers of [(Allan, 1997); (Arnold et al., 2012); (Sreeranjini and Madhukar, 2012a,b)], there after most of the works presented in this dissertation are based on perception like the original works of the ancient mathematicians. However, many research papers, books, a thesis and a dissertation were consulted. We found very few ideas to chip in. Most of ancient mathematics stems from asking the right question. In this dissertation, two questions are addressed: How many algebraic structures are over the aforementioned squares ? And, how many Loub Magic Squares are over the algebraic structures? These questions were set in order to fill the gap of understanding between magic square specialists and modern algebraists.

The statements are: The Tetrabonacci Subgroup of the Symmetry Group over the Loub Magic Squares Semigroup, Rhotrix Subelement Subgroups of the LoubMagic Squares Infinite Additive Abelian Group, Symmetry Group of Odd Lengths over the Loub Magic Squares Semigroup of Multiset of Natural Number, Transformation Semigroup of Odd Lengths over the Loub Magic Squares Semigroup of Multiset of Natural Number, Minimum Zero-Centre Pandiagonal Composite Type II(a) Loub Magic Squares over Multiset of Natural Number Semiring, Loub Magic Squares over Multiset of Rational Number Infinite Field, Composite Loub Magic Squares Infinite Abelian Group Miscellany Case of the Loub Magic Squares Infinite Abelian Group, Type I Composite of Type I Loub Magic Squares Infinite Additive Abelian Group and of Type II Infinite Field, and Concrete Examples of the Minimal Bull Eye Composite Loub Magic Square Infinite Additive Abelian Groups. The above notions are in title style and were what we delved into ( dwelled around). They are what we construct across the board of semigroups, groups, semirings and fields using the Loub Magic Squares and Loub Magic Squares using semigroups, groups, semirings and fields (two ways).

* 1. **Background of the Study**

The following concepts are enough to provide the background for figuring out/ perceiving the theme of this dissertation. The concepts are: Sequence, Semigroup, Group, Fibonacci Sequence, Fibonacci Semigroup, Fibonacci Group, Symmetry Group, Rhotrix Theory, Transformation Semigroup, Finite and Infinite Abelian Group,Semiring, Field and Multiset. Incorporating these concepts with the Loub Magic Squaresas well asits composite inculcate the underlying theme of this dissertation.Why we have to do the research is because originality and creative imagination mark real advances in the realm of Mathematics.

* 1. **Scope and Limitation of the Study**

**Scope:**This dissertation focuses on: How the set of the Loub Magic Squares acquires different structures according to the type of operation defined on it. We studied how the structure of the set is changing as we are increasing the basic algebraic properties of the set and the operation defined on it. Thus, the dissertation dwells around the Construction of Some Algebraic Structures using the Set of Loub Magic Squares.

**Limitation:** This study has not gone beyond multisets of rational number of the forms . Also, it has not delved into the astronomical and the magical applications of the Loub Magic Squares.

* 1. **Aim and Objectives of the Study**

The aim of this dissertation is to prove that Loub Magic Square Algebraic Structure forms a good newest realm of Mathematics.

The aim is achieved through thefollowing objectives.

1. To construct the: Tetrabonacci Subgroup of the Symmetry Group over the Loub Magic Squares Semigroup, Rhotrix Subelement Subgroups of the Loub Magic Squares Infinite Additive Abelian Group, Symmetry Group of Odd Lengths over the Loub Magic Squares Semigroup of Multiset of Natural Number,and Transformation Semigroup of Odd Lengths over the Loub Magic Squares Semigroup of Multiset of Natural Number;
2. To construct: Minimum Zero-Centre Pandiagonal Composite Type II(a) Loub Magic Squares over Multiset of Natural NumberSemiring, Loub Magic Squares over Multiset of Rational Number Infinite Field,Null Magic Sum and Magic Product Bull Eye LoubMagic Squares Ring;
3. To establish: A relationship between Composite Loub Magic Squares Infinite Abelian Group and Loub Magic Squares Infinite Abelian Group, that Loub Magic Squares form a Commutative Semigroup if the underlying set considered is a multiset of natural number, and that LoubMagic Squares form an Abelian Group if the underlying set considered is a multiset of integer number; and
4. To construct the Type I Composite of Type I Loub Magic Square Infinite Additive Abelian Group and of Type II Infinite Field; to concretizevia examplesthe Minimal Bull Eye Composite Loub Magic Square Infinite Additive Abelian Groups; and to construct miscellany algebraic properties of the Lou Magic Squares: Eigen Groups, Centre Piece Groups and Magic Sum Groups.
   1. **Research Methodology**

The ‘logy’ in ‘Methodology’ means ‘study’ just as the one in ‘Biology’ or ‘Logology’. This has to do with methods and the study of the methods. According to some experts in this realm,the abstract algebraic aspect of mathematics, there is no need for methodology. The methods inherited from the ancient times are enough and are as follows: Methods of Proof by Contradiction, Methods of Proof by Counter Example, Method of Proof by Induction Hypothesis and Method of Proof using Propositional Logic. We used the Method of Proof by Counter Example which is use to disprove a universal statement. Though the Method of Proof by Induction Hypothesis is not used directly, yet it is used it indirectly from the fact that we do not go to degree and concluded that we prove mathematics theorem which is a syndrome that affects some parts of developing realms of mathematics. It is because of this method that it is decided not to include 0 in the set of natural numbers. The Method of Proof using Propositional Logic in the general Lou Magic Square is also used.

* 1. **Outline of the Dissertation**

This dissertation contains five chapters as follows:

Chapter 1: Introduction and Preliminaries

Chapter 2: Literature Review

Chapter 3: Methodology

Chapter 4: Findings

Chapter 5: Summary, Conclusion and Recommendations

* 1. **Significance of the Study**

Sets acquire different structures according to the type of operation defined on them. As the number of basic algebraic properties increase, the structures decrease. The structures become more complex, interesting and useful as the basic properties increase. The grouping of these interesting structures is of utmost importance to mathematicians. Both the magic squares and the algebraic structures constructed over them are significant, thus these structures are of double significance. A good stock of examples as large as anything is indispensable for a thorough understanding of any concept and example is the school of mankind and mankind will learn at no other. Concrete examples make realms still stand. The notion of a “group” viewed as the epitome of sophistication, is today one of the mathematical concepts most widely used in Physics, Chemistry, English, and Mathematics itself.

Pythagoras said that number is the origin of all things, and certainly the law of number is the key that unlock the secrets of the universe. There is no science that teaches the harmony of nature more clearly than Mathematics, and the magic squares are like a magic mirror which reflects a ray of the symmetry of the divine norm immanent in all things, in the immeasurable immensity of the cosmos not less than in the mysterious depths of the human mind (Andrews, 2007). The universe is a representation of an internal direct product of the symmetry groups. Be that as it may, we mind the fact that no knowledge is useless and remind mathematicians the definition of “Good Mathematics”.

**1.9.Justification of the Study**

This research serves as a reference work for future researches on the globe for we make numerous recommendations towards the end of the work. John Nashy’s Project is 27 pages, and made him won a Nobel Price (Toby,2014). Galois is the founding father of group theory and his compiled complete work is about 60 pages.The contributions in this dissertation extend to more than 60 pages of Galois. Though number of pages is not the requisite, yet we are contend (joyful) that we did justice to the theme of this dissertation for a lot of research papers can come out of our recommendations. Refer to 5.3 for research directions in this realm.

Only Gauss, the famous mathematician of every century, is both Pure and Applied Mathematician. To the applied thus demagogues: Apply these algebraic structures not only to talisman, building, faith, music, statistical experiment; but also to solutions of system of differential equations (Jacob, 2007)and polynomial equations as like the Symmetry Groups and other Groups.

**1.10.Preliminaries**

We start by presenting preliminary definitions and notions/ terminologies that aid understanding the whole concepts of this dissertation.We start by presenting important definitions (1.10.5, 1.10.7), a refined procedure (1.10.6) and a new generalization (1.10.8) for the Loub**.**

**Definition 1.10.1.** A square formed by removing the border cells of an Loub Magic Square is called the subelement square of the Loub Magic Square.

**Remarks 1.10.2.**Interesting is the least subelement which is a subset of Pancolumn Squares. Purposefully, the Loub Magic Square has no subelement for it is not pancolumn. It is explicated that the subelement squares of the Loub Magic Squares Semigroup form a semigroup and the subelement squares of the Loub Magic Squares Group form a group with respect to the same underlying set and operation.

**Definition 1.10.3.** A basic magic square of order n can be defined as an arrangement of arithmetic sequence of common difference of 1 from 1 to in an square grid of cells such that every row, column and diagonal add up to the same number, called the magic sum M(S) expressed as and a centre piece c as **.**

**Definition 1.10.4.** Main Row or Column is the column or row of the Loub Magic Squares containing the first term and the last term of the arithmetic sequence in the square**.**

**Definition 1.10.5.** A Loub Magic Square of type I is a magic square of arithmetic sequence entries such that the entries along the main column or row have a common difference and the main column or row is its central column or central row**.**

**1.10.6 Loub Procedure (NE-W-S or NW-E-S, the Cardinal Points).** Consider an empty square of grids (or cells)**.** Start, from the central column or row at a position where is the greatest integer number less than or equal to x, with the number 1**.** The fundamental movement for filling the square is diagonally up, right (clockwise or NE or SE) or up left (anticlockwise or NW or SW) and one step at a time**.** If a filled cell (grid) is encountered, then the next consecutive number moves vertically down ward one square instead**.** Continue in this fashion until when a move would leave the square, it moves due N or E or W or S (depending on the position of the first term of the sequence) to the last row or first row or first column or last column; where NE stands for North-East and NW, S, E, S, SW, etc. stand for the analogues and n is an odd natural number greater than 1.

**Definition 1.10.7.** Loub Magic Squares of type II are magic squares constructed with the Loub Procedure of repeating- pattern- sequence**.**

**1.10.8. The Generalized Loub Magic Square**

Let denotes the set of integer number, denotes the ‘exclusive or’ and denotes the ‘inclusive or’**.** Then the general Loub Magic Square is given by

here denotes the miscellany effects of rotations and\or reflections ofLoub Magic Square and denotes the composition ofLoub Magic Square S**.**

The advantage of this generalization is that it covered both miscellany effects and Composite Loub**.** With rotations and/or reflections, a single Loub Magic Square will give 7 miscellany effects, and are covered in the generalization**.** For the effects, see (Gan *et al*., 2012)**.**

**1.10.9. The Proof of the and of the , Where**

**Theorem 1.10.10.** Let the arithmetic sequence be arranged in an Loub Magic Square. Then the magic sum of the square is expressed as and the middle term of the sequence (centre piece of the square) is expressed as where denotes the common difference of entries along the main column or row and is given as .

**Proof.** Consider any arbitrary General Loub Magic Square (here we consider ) as follows:

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Let and . Then we have (from the square) an arithmetic sequence: having the sum as

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i.e. and from the Gaussian High School (Elementary) Method.

Since our square is , m number of cells (terms) are on the main column whence

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Thus, (3) and (4) become … (5) and respectively. And,

where is along the main column.

Substituting (7) in (6), we have: … (8).

Substituting (8) in (5), we get:

From (3) and (4), ,

i.e. . Comparing (9) and (10), we have:

**Definition 1.10.11.** A non-empty set S (or G)equipped with a binary operation \* is said to be a Semigroup (S,\*) if it satisfies the following axioms:

1. and

If in addition to the 2 axioms above, the following 2 axioms are satisfied; then we call the algebraic structure a group.

1. .

If in addition to the above 4 axioms: i; ii; iii; and iv; the following is also satisfied; then we call (G,\*) an abelian group.

1. .

**Remark1.10.12.** The shift in notations from the use of S to G by the respective specialists is intentional and the closure property is already implied by the binary operation, it is included because it is often overlooked.

**1.10.13. The Generalized (Symbolized) and Squares**

Normally, symbols represent unknown number. Where known number is symbolized is where generalization of number of such a sort is of demand. The following result means such a generalization on magic squares. The result is of interest for a general magic square has taken care of the rotations and/or reflections, with variation of symbols, of the square, and the general Loub Magic Square has no general Loub Magic Square Subelement.

**Conjecture 1.10.14.** Generalization (Symbolization) of magic squares does a magic of generalizing all the values of the common difference of the arithmetic sequence as well as the generalized Loub Magic Square has, up to the least subelement, no general Loub Magic Square.

This classicallexicographic abstraction (symbolizing number) is meaningfully significant.

Let c be the middle term of the arithmetic sequence arranged in an Magic Square with the Loub Procedures (NE-W-S or NW-E-S, the Cardinal Points), let be the common difference of the sequence, be the common difference of the sequence along the main column and is along the main row. Then the generalized 3 Loub Magic Square is available in (Lee, 1986) and we make a little adjustment to have:

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here and the generalized Loub Magic Square is

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This section is concluded by showing that the generalized Loub Magic Square is not a least subelement of the generalized Loub Magic Square even though

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f and only if seemingly. The subelements of such sorts (types) are pancolumn.

* + 1. **. The Loub Magic Squares Semigroup and Group**

We hereby present that the set of the Loub Magic Squares over set of natural number equipped with the binary operation of matrix addition forms a semigroup, and over the set of integer number forms a group-enclosed with the same operation.

Consider arbitrarily 3 general Loub Magic Squares:

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Lee(1986) provide the idea of the generalized Loub Magic Square.

We now show that they satisfy the **two axioms of semigroup** as follows:

i.**Closure**. Let . Then:

ii. **Associativity**. Since we are dealing with number as entries of the square, then:

We showcase the **two other axioms** of the group as follows:

iii. Since we underlie set of integer number here, then ; a singleton set of the arithmetic sequence 0, 0+0, ...,0+0+0+0+0+0+0+0+0; can be arranged in the square to form an **identity element** as follows:

It is clear that if a is a Loub Magic Square, then

iv. Let us arbitrarily choose the following Loub Magic Square . Then, its **inverse** is , i.e.

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We can now consider the general multiset. The Loub Magic Square, over the multiset of integer number since multiset of natural number is its subset, is semi pandiagonal. By semi pandiagonal, we mean in an square, n elements repeat on every row, column and on a diagonal. Though the sum of the number on the rows, the columns and the diagonals add up to the magic sum; yet one diagonal has an n repetition of one element. To change the orientation (from left to right or the reverse) of the pandiagonal of the , use the sequence ; see (Babayo and Garba, 2015k).

We can now show that they form a group as in the above. Consider 3 arbitrary elements of the Lefty Semi Pandiagonal Loub Magic Squares:

Then:

i.

is also a Lefty Semi Pandiagonal Loub Magic Squares, hence **Closure Property** is satisfied.

ii. **Associativity Property.**It is clear (even from inherited property of the underlying set) that

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iii. The **identityelement**is as in the above

where each zero(except the first entry of the square) is a series of zeros correspondingly n times.

1. Let the following be an arbitrary Lefty Semi Pandiagonal Magic Square

Clearly, its **inverse** is

v. Every 2 Loub Magic Squares (whether semi pancolumn or not) over multiset of natural number or over multiset of integer number commute since natural and integer numbers commute.

Thus, the group and the semigroup of the Loub Magic Squares are **commutative**.

* + 1. **Centre Pieces and Magic Sums Abelian Groups**

**1.10.16.1. Centre Pieces Abelian Group**

The set of the centre pieces of Loub Magic Squares equipped with the integer number binary operation of addition “+” forms an infinite abelian group. Given the centre pieces of Loub Magic Squares with corresponding formula(s),

1. is the centre piece of the Loub Magic Square with first term and common difference along the main column . Hence, the set is **closed**.
2. **Associativity Property.**This is an inherited property of :
3. The **identity element** is the zero centre piece e.g. .
4. Given an arbitrary centre piece of the Loub Magic Square, thenthe centre piece of the Loub Magic Square having first term as and common difference along the main column or row as ,whose formula is such that , the identity centre piece.

Thus, are **inverses** of each other.

1. **Commutativity Property.**

Clearly,

The aforementioned set equipped with the operation is an **abelian group**.

**1.10.16.2. Magic Sum Abelian Groups**

The set of the magic sums of Loub Magic Squares equipped with the integer binary operation of addition form an infinite abelian group.

Given the magic sums of Loub Magic Squares with corresponding formula:

hen (as in the above):

1. Loub Magic Square with first term and common difference along the main column

The axioms: follow, by analogy to the centre piece abelian group, immediately.

**1.10. 16.3.Eigen Values Abelian Group**

The eigen values computation in the magic squares is what is zealotly prophesized that magic squares are special type of matrices, hence the definition of the magic squares, we do not love to like such a sudden conclusion if loving liking forces choosing the definitions in terms of the square grids (or cells).

We want to show through concrete examples that the set of Eigen Values of the Loub Magic Squares equipped with the usual binary operation of integeraddition forms a group. Consider the following arbitrary two Loub Magic Squares which we let

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We compute the eigen values for as follows: The corresponding matrix of is and its characteristic equation is

having eigen values .

We compute the eigen values for b as follows: The corresponding matrix of b is , its characteristic equation is

i.e.

with eigen values

We compute the eigen values for c as follows: The corresponding matrix of c is and its characteristic equation is , i.e. with corresponding eigen values .

We now conclude this section by showing that the set of eigen values satisfies **The Properties of a Group** as follows:

**Closure Property.** Consider any 3 arbitrary Loub Magic Squares a, b, c (say); such that ; then from the example above, the corresponding eigen values of a; the corresponding eigen values of b; are such that where are the corresponding eigen values of c.

**Associativity Property.** Since the set of Loub Magic Squares is a semigroup (which is easy to observe), the binary operation are associative.

**Identity Element Property.** The eigen value 0 is the identity element that corresponds to the sum of the Loub Magic Squares of opposite eigen values as in the above.

**Inverse Elements Property.** For any arbitrary eigen value corresponding to a Loub Magic Square m, there exists a eigen value corresponding to another Loub Magic Square such that gives the identity element which is formed as a result of matrix addition of the aforementioned Loub Magic Squares.

**Commutativity Property.** Consider any two arbitrary Loub Magic Squares a and b ; then; then from the example above, the corresponding eigen values of a; the corresponding eigen values of b; are such that .

The idea of eigen values computation of a magic square is conceived from the work of (Daryl, 1993).

* + 1. **The Subelement Squares Semigroup and Group**

The set of least subelement of Loub Magic Squares is a subset of Pancolumn Squares. By convention, the Loub Magic Square(since not pancolumn) is not a self subelement, but is the least subelement magic square of the Composite. The sum of two arbitrary subelements of Loub Magic Squares is a subelement of Loub Magic Square, hence closure is exhibited. Associativity , Identity, Inverse and Commutativity Properties are inherited from the super elements, the set of Loub Magic Squares. Both the binary and the unary operations of the super elements and of the subelements are equal.

**CHAPTER TWO**

LITERATURE REVIEW

**2.1.Background Works**

Magic Square has been investigated over 4120 years (Anup *et al.*, 2012). Mathematicians introduced it long time before astronomers picked up interest in it. Magic Square is lost in mathematics and the lossis attributed to astronomers for many centuries. Recently, mathematicians tend to bring it back to Mathematics again after arguably semigroup, group and field realms have gone far. The oldest magic square recorded is the Lo Shu Magic Square. Itoriginated from China. The tradition has it that the emperor Yu was standing at the edge Shu of the river Lo (approximately 2200 BC) when a tortoise appeared with the square upon its back. De La Loub, a French diplomat, devised a method of constructing the Lo Shu Magic Square in his book: “A New Historical Relation of the Kingdom of Siam” under a chapter titled, “ The Problem of the Magical Square According to the Indians”.

The exact origin of magic square is unknown, but most scholars believed it originated from China. It was originally meant to do magic, music, art, talisman, statistical experiment and witchcraft.

Anton Kazimirovish Suschkewitsch is the world’s first semigroup theorist(Christoper, 2009). He is born in Borisoglebsu, Russia and attempted a systematic study of various classes of generalized groups of which semigroup is inclusive in his Ph.D. Dissertation, “The Theory of Operations as the General Theory of Groups” in the Department of Mathematics, Kharkov, Ukraine in 1917. It was said that Suschkewitsch met Emmy Noether, who noetherian ring are defined after her, in Berling. Rees andClifford contributed enormously to the realm. Rees constructed a semigroup which is now known as Rees Matrix Semigroup, a semigroup constructed with matrices and is a partial generalization of the triples. Clifford constructed a semigroup, the Clifford Semigroup(Rucuc, 2000), which seemingly looks very close to idempotency.

Henrich Brandt in 1927 wrote a research paper containing the notion of groupoid, a semigroup without associativity and an algebraic structure which arose from Brandt’s work on quaternary quadratic forms, which he set out to generalize some results of Gauss(1777-1855), the famous mathematician. He demonstrated that for a given discriminant, the classes of primitive integral binaryquadratic forms have the structure of a finite abelian group, the group of Neil Henrik Abel (1802-1829)[(Christopher, 2009) and (Jonathan, 2008)].Galois (1811-1832) was born near Paris and was recorded as the first to introduce the concepts: Group, normal subgroups, isomorphism, simple groups and finite fields. Galois entire collected works full only 60 pages. Then Ajibade(2003), a Nigerian Mathematician, introduced the finest notion, Rhotrix Theory. History has a habit of repeating itself.

**2.2. Literature View I**

Arguably, the first set of recorded works that are related to our conception of magic squares algebraic structure are [(Sreeranjini and Madhukar, 2012a,b)]. We conceived the idea of miscellany effects of magic squares from the work of Lee(1986) and the definition of Composite Magic Square is in the work of Ahmed(2004). The concept of eigen values computation is in the work of (Daryl, 1993), the concept of Fibonacci Sequence is in Joseph(2005), the concept of Fibonacci Semigroup and Group is inUmar(2012), the idea of Symmetry Group is in Vasistha and Vasistha (2006), the idea of Transformation Semigroup is in Rucuc(2000) and the idea of a generalized magic square is in(Lee, 1986).

**2.3. Literature Review**

The works of (Sreeranjini and Madhukar, 2012b) contained egregious error for they ended up proving that a constant is a field rather than the semi magic constant of the magic squares and(Sreeranjini and Madhukar, 2012a) lack an idea of a semi magic constant of a magic square plus a semi magic constant of a magic square is equal to a semi magic constant of another magic square. Finally, they ended up dwelling around a semi magic square containing entries only in its main diagonal, where all other entries

they considered were zeros. Had it been the literature was well presented, a work similar to this one most has been done earlier.

Gan *et al.* (2012) cited the 7 miscellany effects of rotations and/or reflections of Type I Loub , but had no idea of the Type II Loub much less its miscellany effects. Frankly, we constructed the miscellany effects originally with clockwise and anticlockwise motions before coming across the work of (Gan et al.,2012).

The work of (Daryl, 1993) is a babyish tautology even from the title of the work if really magic squares are special type of matrices. Daryl(1993) has an idea of eigen values computation of magic squares, but never attempted to make comparison of eigen values of a magic square with another much less an idea of eigen group.

Vasistha and Vasistha(2006) and Joseph(2005) had no idea of Loub Magic Squares Semigroup, Group or Field, and(Nikola, 2000) had no idea of Transformation Semigroup over magic square of any sort (type) and presentation is also ordered pair as like the binary relation, Ree’s Matrix Semigroup, Rectangular Band, Bicyclic Semigroup, and lots more. It is also a generalization job.

Our generalized magic square is gotten from (Lee, 1986) though we made a little adjustment to arrive at our own. We however used the idea of the construction of the to construct the generalized . Our set of generalized is original for we have not meet a literature on our type II Loub Magic Square (satisfying Loub Procedure). This is the reason for settingout revisionism of the definition of squares in general. It is proposed that the definition of magic square has to carry along its main aim.

**2.4.Our Contribution to Loub Magic Squares: Miscellaneous Properties of the Loub Magic Squares**

Miscellany Properties of the Loub Magic Squares are concretized via examples. This work outlinesvia concrete examples that apt for generalizations the generalized (symbolized) and squares, the algebraic development of eigen values and magic sums of the square, the underlying sets under discussion of the squares, the rhotrix-matrix construction of the square, introduction of the terminology: Subelements of the squares, partitioning the squares, the proof of the generalized centre piece of the square as and of the magic sum M(S) as where is the first term, is the common difference along the main column expressed as , and is the last term of the arithmetic sequence, the proof of the square is magic when m is odd including the primes except when m is the oddest (even) prime, and an example of the Cayley Table Magic Square. is the greatest natural less than or equal to x.

**Definition 2.4.1.** A basic magic square of order n can be defined as an arrangement of arithmetic sequence of common difference of 1 from 1 to in an square grids of cells such that every row, column and diagonal add up to the same number, called the magic sum expressed as and , where is the greater natural less than or equal to x.

A basic magic square can be rotated and/or reflected to give 7 other magic squares that are considered Loubeffects. Symbolizing the Loub Squares is like moving to greater abstraction, but has generalize both the rotations and/or reflections. So far, the simplest proof is a mere intimate curious observations of the arithmetic sequence in the square. Refer to (Gan *et al.*, 2012) for the miscellany effects of rotations and/or reflections of the basic magic square. We give 2 examples one for each of the generalized and squares. And, a conjecture that stimulates provision of a generalized result is presented.

Given any 3 arbitrary Loub Magic Squares such that the sum of corresponding entries of 2 chosen squares will give corresponding entries of an alike positioned cells in the 3rd one; then the set of magic sums and the set of eigen values form respective abelian groups. This result is apt for a simple generalization.

Before any algebraic structure is studied/ investigated, an underlying set is firstly so considered and firstly so, we consider the multiset of natural numbers followed by the multiset of integer numbers and purposefully propose that the multisets of some rational, some real and some complex numbersare beyond the scope of this work.

Applause is to the finest founders of one of the newest realm of mathematics; the rhotrix theory which has to do with the rhomboidal array of number; and so far, its applications are seemingly very rare or verbatim virtuoso of the matrix theory applications. A juxtaposition alike of the construction of magic squares is coiled with an infix adjoinment of zeros to be referred to as the rhotrix-matrix construction for there exists an inter switch of matrix and rhotrix transformation system which forces to provide the magic squares. It is set to provide the concrete examples of and , a provision that can be generalizednoting that square is isomorphic to one of the chosen aforementioned underlined set.

**Definition 2.4.2.** Main Row or Column is the column or row of the Loub Magic Squares containing the first term and the last term of the arithmetic sequence in the square.

The notion of subelements and the study of properties and inter properties of the squares subelements of the Loub Magic Squares are introduced. The most paramount importance of this lies in the fact that a method of construction of basic squares that are mostly pancolumn and that have no already known method of their construction earlier is got. A study of the relationship of the least subelements () of the is done and is followed by a recommendation for ageneralization.

Given any Loub Magic Square having entries an arithmetic sequence from to with a common difference of along the main column, a proof that the magic sum M(S) is expressed as and the centre piece (middle term of the sequence c) as where are provided.

It is established through counter example that for all Loub Magic Squares where m is odd including the primes, m cannot be the oddest (even) prime.

We also give an example of the Cayley Table Magic Square, a result that has a generalization.

**2.4.3.The Generalized (Symbolized) and 5 Squares**

Rarely do symbols represent unknown number and where known number are symbolized is where generalization of number of such a sort (type) is of demand. The following result means such a generalization on magic squares. The result is of interest for a general magic square has taken care of the rotations and/or reflections, with variation of symbols, of the square, and the general Loub Magic Square has no general Loub Magic Square Subelement.

**Conjecture 2.4.4.** Generalization (Symbolization) of magic squares does a magic of generalizing all the values of the common difference of the arithmetic sequence as well as the generalized Loub Magic Square has, up to the least subelement, no general Loub Magic Square.

This classical lexicographic abstraction (symbolizing number) is meaningful and significant.

Let c be the middle term of the arithmetic sequence arranged in an Magic Square with the Loub Procedure (NE-W-S or NW-E-S, the Cardinal Points), let be the common difference of the sequence, be the common difference of the sequence along the main column and is along the main row. Then the generalized 3

Loub Magic Square is available in (Lee, 1986) and a little adjustment is made to have:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

here and the generalized Loub Magic Square is

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

This section is concluded by showing that the generalized 3 Loub Magic Squares is not a least subelement of the generalized Loub Magic Square even though

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

s isomorphic to

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

f and only if seemingly. The subelements of such a sort (type) are pancolumn.

**2.4.5.The Underlying Sets that Outline Algebraic Structural Properties of the Loub Magic Squares**

For the set of Loub Magic Squares with respect to addition operation to form a semigroup, the underlined sequence has to be of the multiset of natural number. If in this multiset, the zero element is inclusive; then the squares form a semigroup with identity. The binary operation of addition enclosed or equipped is of the set of natural number.

The set , which is considered as a special case of the scalar multiplication over the Loub Magic Squares, forms an abelian group also. So far, this as well as the sets of some rational, some real and some complex number(s) are beyond the scope of this work.

In the Loub Magic Squares, it is important to note that the underlying multiset elements maintain order. That is, which is also in line with the definition of multiset.

**2.4.6.The Rhotrix-Matrix Construction of the Loub Magic Squares**

A rhotrix is a rhomboidal array of number and a matrix is a rectangular array of number. Usually, the rhotrix less the matrix of equal dimension with a multiple of 3, number of elements and all rhotrices are of odd dimensions. So far, the applications of rhotrices, unlike matrices, are seemingly very rare or usefully verbatim virtuoso of matrix applications. For more details of rhotrix theory, refer to the works of Ajibade (2003) and Mohammed(2011).

We are set to firstly so provide the construction of for the square is isomorphic to any chosen underlined set. Then it is followed by the construction of

**2.4.7.The Construction of the Squares: An Example**

Consider an arbitrary arithmetic sequence: 1,2,…,9. Write the sequence into a square matrix as follows:

Transform the matrix to a rhotrix as follows:

Swap the row with row and column with column (elementary row and/or column operation alike).

Overlap the row into the row and the row into the row , and overlap the column into the column and the column into the column to transform the rhotrix back to matrix again as follows:

Apply the square grid of cells as follows:

|  |  |  |
| --- | --- | --- |
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

This is a Loub Magic Square.

This work recommends provision of the rhotrix- matrix construction of the Loub Magic Square.

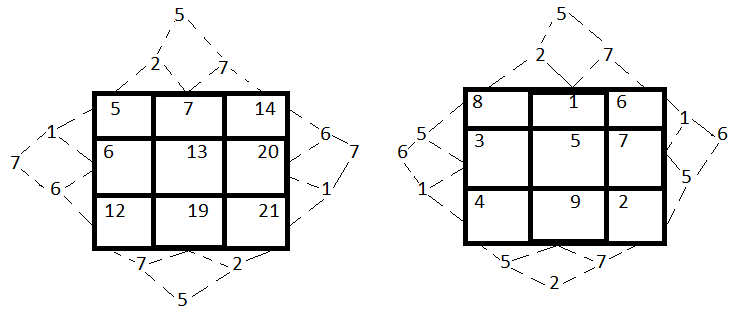
**2.4.8.Properties of the Subelement Squares of the Loub Magic Squares**

Obviously, the least subelement of the aforementioned squares are the Squares.

Let d be the common difference of the arithmetic sequence in the square. Then we denote the set of all Loub Magic Squares by .

**Property 1.** is not a self least subelement for it is not pancolumn. We propose this to avoid ambiguity.

**Property 2.** There exists an interesting addition juxtaposition between and the least subelement of as follows:

****

**Definition 2.4.9.** A diagonal having common difference d of entries in is referred to as the leading diagonal.

**Property 3.** Every least subelement has a leading diagonal.

**Property 4.** The set of all least subelements is a subset of the pancolumn squares.

**Property 5.**There exists a rhotrix subelement of every Loub Magic Square.

**2.4.10.Partitioning the Loub Magic Squares Set**

Having denoted the set of all the Loub Magic Squares by L[d, ] where d is the common difference of the arithmetic sequence in the square, the variable values of d partitioned into infinitely many distinct equivalence classes.The following theoremis demonstrated via concrete examples.

**Theorem 2.4.11.** The common difference d of an arithmetic sequence in the Loub Magic Squares set L[d, ] partitioned it such that , .

**Proof.** partitioned into disjoined equivalence classes, and if ,then.

**Remarks 2.4.12.Concrete Demonstration of the**

Recall that given a magic square

|  |  |  |
| --- | --- | --- |
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

, there exists 7 miscellany effects of the rotations and/or reflections of it. This showcases that corresponding to every element of are 7 other rotations and/or reflections of it. This implies that to every equivalent class, there corresponds 7 other equivalenceclasses.

The following are examples of the partition: where....

**2.4.13.The Proof of the Square of Arithmetic Sequence 1 to where m Is Odd and/or Prime Is Magic Except When m Is the Oddest (even) Prime**

Providing a counter example is an enough proof to show that the Magic Square of the arithmetic sequence 1, 2, 3, 4 is not a magic square.

Having 4 consecutive objects in the Square, we have 4!=24 possible arrangements as follows:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  | | --- | --- | | 1 | 2 | | 3 | 4 | | |  |  | | --- | --- | | 1 | 2 | | 4 | 3 | | |  |  | | --- | --- | | 1 | 3 | | 4 | 2 | | |  |  | | --- | --- | | 1 | 3 | | 2 | 4 | | |  |  | | --- | --- | | 1 | 4 | | 2 | 3 | | |  |  | | --- | --- | | 1 | 4 | | 3 | 2 | | |  |  | | --- | --- | | 2 | 1 | | 3 | 4 | | |  |  | | --- | --- | | 2 | 1 | | 4 | 3 | |
| |  |  | | --- | --- | | 2 | 3 | | 4 | 1 | | |  |  | | --- | --- | | 2 | 3 | | 1 | 4 | | |  |  | | --- | --- | | 2 | 4 | | 1 | 3 | | |  |  | | --- | --- | | 2 | 4 | | 3 | 1 | | |  |  | | --- | --- | | 3 | 1 | | 2 | 4 | | |  |  | | --- | --- | | 3 | 1 | | 4 | 2 | | |  |  | | --- | --- | | 3 | 2 | | 1 | 4 | | |  |  | | --- | --- | | 3 | 2 | | 4 | 1 | |
| |  |  | | --- | --- | | 3 | 4 | | 1 | 2 | | |  |  | | --- | --- | | 3 | 4 | | 2 | 1 | | |  |  | | --- | --- | | 4 | 1 | | 2 | 3 | | |  |  | | --- | --- | | 4 | 1 | | 3 | 2 | | |  |  | | --- | --- | | 4 | 2 | | 1 | 3 | | |  |  | | --- | --- | | 4 | 2 | | 3 | 1 | | |  |  | | --- | --- | | 4 | 3 | | 1 | 2 | | |  |  | | --- | --- | | 4 | 3 | | 2 | 1 | |

**Remark 2.4.14.**Other possibilities can give magic squares, but never Loub

**2.4.15.Cayley Table Magic Square: An Example**

Consider the binary operation \* defined on the set as x\*y is the remainder when is divided by 16. Then forms a group (easy to check) with the Cayley Table as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| \* | 1 | 3 | 9 | 11 |
| 1 | 1 | 3 | 9 | 11 |
| 3 | 3 | 9 | 11 | 1 |
| 9 | 9 | 11 | 1 | 3 |
| 11 | 11 | 1 | 3 | 9 |

The table gives an idea of a relationship of a finite group and the magic squares as well as reflecting our underlined sets, the two multisets.

**2.5. Literature View II**

This pioneer work disclosed new realm of semigroups, groups, semirings and fields, the Loub Magic Square Semigroups, Groups, Semirings and Fields. The set of the Loub Magic Squares of the arithmetic sequence of the set of natural number or of its multiset form a semigroup which by analogy we refer to as the Loub Magic Squares Semigroup; and the set of the Loub Magic Squares of the arithmetic sequence of the set of integer number or of the multiset of the integer number form a group which by analogy we refer to as the Loub Magic Squares Group. The aforementioned semigroup and group are both with respect to the matrix binary operation of addition.

The collection of the centre pieces with formula equipped with the integer operation of addition forms an abelian group and the set of all the magic sums with formula equipped with the integer number binary operation of addition forms an abelian group also, where and are the corresponding first term and common difference along the main column respectively of Loub Magic Squares. We also showcase that the set of eigen values of the Loub Magic Squares enclosed with the integer number operation of addition forms an abelian group. This is meaningful as the principal value of the eigen values corresponds to the magic sum, see (Daryl, 1993).

Miscellany Properties of the Loub Magic Squares are concretized via examples, see also (Babayo and Garba, 2015b). This work outlinesvia concrete examples that apt for generalizations the generalized (symbolized) and squares, the underlying sets under discussion of the squares, the rhotrix-matrix construction of the square, introduction of the terminology: Subelements of the squares, partitioning the squares, the proof of the generalized centre piece of the square as and of the magic sum M(S) as where is the first term, is the common difference along the main column expressed as , and is the last term of the arithmetic sequence, the proof of the square is magic when m is odd including the primes except when m is the oddest (even) prime, and an example of the Cayley Table Magic Square.

A basic magic square of order n can be defined as an arrangement of arithmetic sequence of common difference of 1 from 1 to in an square grid of cells such that every row, column and diagonal add up to the same number, called the magic sum expressed as and ,( where is the greatest natural less than or equal to x).A basic magic square can be rotated and/or reflected to give 7 other magic squares that are considered Loubeffects. Symbolizing the Loub Squares is like moving to greater abstraction, but has generalized both the rotations and/or reflections. So far, the simplest proof is a mere intimate curious observation of the arithmetic sequence in the square. We refer to (Lee , 1986) for the miscellany effects of rotations and/or reflections of the basic magic square, and we give 2 examples one for each of the generalized and squares. And, a conjecture that stimulates provision of a generalized result is presented.

Given any 3 arbitrary Loub Magic Squares such that the sum of corresponding entries of 2 chosen squares will give corresponding entries of an alike positioned cells in the 3rd one; then the set of magic sums and the set of eigen values form respective abelian groups. This result is apt for a simple generalization. Before any algebraic structure is studied/ investigated, an underlying set is firstly so considered and firstly so, we consider the multiset of natural number followed by the multiset of integer number and purposefully propose that the multiset of some rational, some real and some complex numbersare beyond the scope of this work.

Applause is to the finest founders of one of the newest realm of mathematics, the rhotrix theory which has to do with the rhomboidal array of number; but so far, its applications are seemingly very rare or verbatim virtuoso of the matrix theory applications. A juxtaposition alike of the construction of magic squares is coiled with an infix adjoinment of zeros to be referred to as the rhotrix-matrix construction for there exists an inter switch of matrix and rhotrix transformation system which forces to provide the magic squares. We are set to provide the concrete examples of and , a provision that can be generalizednoting that square is isomorphic to one of the chosen aforementioned underlined set.

A square element removed from an element of the set of all the Loub Magic Squares is called the subelement square and Main Row or Column is the column or row of the Loub Magic Squares containing the first term and the last term of the arithmetic sequence in the square. We introduce the notion of subelements and study the properties and inter properties of the square subelements of the Loub Magic Squares. The most paramount important of this lies in the fact that a method of construction of basic squaresthat are mostly pancolumn and that have no already known method of their construction earlier is got. The relationship of the least subelements () of the is studied. It can also have a generalization.

In this work, the permutation and its composition over the Loub Magic Squares Semigroup is logically introduced to form a subgroup of the Symmetry Group which by analogy to the Fibonacci Group is termed the Tetrabonacci Group. The concepts of Fibonacci Semigroups (Umar, 2012) as well as theconcepts of Symmetry Groups (Joseph, 2005) are widespread in the Modern Algebra Literature. The underlying sequenceconsideredherein the aforementioned magic squares is the arithmetic sequence with unity common difference. This does not sidetrack generalization, and the generalization of the general concepts revealed in this work. A conjecture is set.

In the permutation of the Symmetry Group, only the final effect is recognized; but for the construction of the aforementioned subgroup, both the motion which may be considered as clockwise + NE-W-S or anticlockwise + NW-E-S Loub Procedures of construction where NE stands for North-East, W stands for due West, S stands for due South, and NW and E stands forthe analoguesand the effect which has to do with the final result of disjoint cycle elements of the set of permutation. This is of double interest for magic squares by themselves are energizing and important. See the work of (Sree, 2005) for some of such stimulations.

With the composition of the maps on the 4 elements per the general 8 miscellany effects [Gan *et al.,* 2012) of rotations and/or reflections of 1 Loub Magic Square defined on the set of their permutations, the Tetrabonacci Groupan idea analogous to the Fibonacci Groupis formed. The reason for such brevity is not farfetched**.**

In this work, permutation over the Loub Magic Squares enclosed with its composition forms the Symmetry Group of odd lengths. Not only to generate Symmetry Groups of odd lengths with the aforementioned squares is the aim of this work, but to minimize the number of the squares that generates it. Given a Loub Magic Square, there are 7 miscellany effects (Gan *et al*., 2012) formed as a result of rotations and/or reflections of it if the underlying set is the set of integer number, but if it is its multiset, 4 miscellany effects are recorded for 3 are missing as a consequence of isomorphism.

If a permutation is defined on the collection of the squares and their effects, the collection of the maps and its composition forms the Symmetry Group of odd length. The odd length is because the permutations are over the Loub Magic Squares and Loub Magic Squares are of the form where n is odd except , the triviality.It will be remarkable if the number of Loub Magic Squares is minimized. This is also an aim of this work. To economize space, we showcased that this is true for Symmetry Group of length 3 and no deep mathematics is required to make such a sudden conclusion of generality.

The job here, however along with caution of the Cayley Theorem, is to find the minimum number of the Loub Squares that will completely cover the structure of all the elements of the Transformation Semigroup of odd lengths partial map. If the Loub Magic Square subsets are able to construct both the permutation subsets of the all together maps from a set to itself and both the maps form respective Symmetry Group and Transformation Semigroup(Howie, 2003) with respect to the composition of maps and the set of the Loub Magic Squares equipped or enclosed with matrix binary operation of addition is an abelian group, then the Cayley Theorem for both the group and the semigroup still stand while all the job joyously jugs for the subsets of the squares so considered are not groups or semigroups by themselves.

We know it may be exactly possible we can define elementary row and column operations on a Loub Magic Squares to construct the Transformation Semigroup, but never attempted it for the higher the n in transformation semigroup , the more cumbersome is such a set of all such maps that are apt to be equipped with composition to form it. The order of and the order of , and In this work, it is showcased that the number of the Loub Magic Square mappings will generate number of all maps to be equipped with composition to form the transformation semigroup. To economize space, we present a concrete example of the since Loub Magic Square is a triviality and does not exist. The n has to always be odd matching the exact definition of the Loub Magic Square.

The Rhotrix Subelements of the square are the rhomboidal array of number that are sitting in the magic squares. The matrix subelements have analogous meaning and are considered babyish tautology, but we need the 2 arrays together to get the method: Rhotrix-Matrix Construction of the Loub Magic Squares. This implies that one of the newest realm of mathematics, the Rhotrix Theory has now gotten another genuine application. We showcase via concrete examples that the Rhotrix Subelements of the aforementioned squares form infinite additive abelian group. The job here are two: First, to present the Rhotrix-Matrix Construction of and Magic Squares where the is Loub The construction of Magic Squares via Rhotrix-Matrix Method is one of the finest achievement of one of the newest realm of mathematics, the Rhotrix Theory first introduced by (Ajibade, 2003). Secondly, we explicate that the Rhotrix Subelements of the Loub Magic Squares form an infinite additive abelian group. For more details of abelian groups, see (Vasistha and Vasistha, 2006) and (Joseph, 2005).

This work is pioneering investigation of the Loub Magic Squares over multiset of rational number of the form infinite algebraic field, where denotes the multiset of integer number. Let L denotes the set of Loub Magic Squares. It is explicated that the set equipped with the matrix binary operation of addition (as we denote it) forms an infinite additive abelian group, and the set enclosed with the rational number multiplication (as we denote it) forms an infinite multiplicative abelian group if the underlying set so considered of the entries of the aforementioned squares is the multiset of the aforementioned set of number. forms an infinite field. The Loub Magic Squares over the multiset of rational number of the form forms an infinite multiplicative abelian group,, thus, making an infinite field, see also (Babayo and Garba, 2015a). This is not the explication of the definition of the field presented in (Sreeranjini and Madhukar, 2012a), but the two are analogous.

Arbitrarily, Semi Pandiagonal Loub Magic Square is considered not squares such that to economize space, and not such a square to dodge near bias choice. The multiset of rational number of the form is aptly considered even though the multiset of rational number of the form, a special type of scalar multiplication, will do. Also, we consider the sequence than a, b, c, ... n times, a, b, c, ... n times, ... though the later will also give an analogous result; but presenting both the two is also a babyish tautology.

In this work, mystic miscellaneous algebraic properties of the set of Composite (Nested) Loub Magic Squares are vividly visualized, see also (Babayo and Garba, 2015j). And, verbatim virtuoso of algebraic properties of the Loub Magic Squares viz: Eigen Group, Magic Sum Group and Centre Pieces Group elegantly viewed the algebraic properties of its Composites. It is also showcased that both the 2 sets equipped with the binary operation of matrix addition form infinite additive abelian groups.It is remarkable that almost joyously the sets of eigen values, centre pieces and magic sums of the Loub Magic Squares Infinite Abelian Group form Infinite Additive Abelian Groups. For magic squareeigen value computations, see (Daryl, 1993). We highlighted consortium of miscellany effects of rotations and/or reflections (Gan *et al*., 2012) and/or enumerations of the Loub Magic Squares to figure out the consortium of the Composites. Establishing such a fact relationship set us conjecture that the Composite (Ahmed, 2004) Loub Magic Squares Infinite Additive Abelian Group is a miscellany case of the Loub Magic Squares Infinite Abelian Group.

By Type I Composite of Type I Loub Magic Squares, we understand the set of all Type I Loub Magic Squares such that each cell (grid) of its element is a Type I Loub Magic Square, the magic squares formed by the De La Loub Procedure over the arithmetic sequence of common difference of 1. In this pioneering work, it is explicated that the aforementioned set equipped with the matrix binary operation of addition(as we denote it) forms an infinite additive abelian group as well as we showcased that the set of Type I Composite of Type II Loub Magic Squares, the magic squares constructed with the Repeating-Pattern-Sequence Loub Procedure, equipped with the aforementioned operation and with the rational number multiplication (as we denote it) forms an Infinite Field if the underlying multiset of entries of the squares is of the rational number of the form. forms an infinite field.

The set of Loub Magic Squares of type I equipped with the binary operation of matrix addition forms an infinite additive abelian group if the underlying set is of the multiset of integer number, and the set of Loub Magic Squares of type II equipped with the matrix binary operation of addition and with the rational number multiplication forms an infinite field if the underlying multiset is of integer number including some rational number of the form . The Minimum Zero-Centre Pandiagonal Composite Type II(a) Loub Magic Squares over Multiset of Natural Number Semiring is also delved into, see also (Babayo, Garba and Khan, 2015).

It is remarkable if the composite of the aforementioned types of magic squares form the aforementioned respective groups and fields. This is our proposed construction, and however along with the scope, a simple definition, a finer procedure and a newest generation of the Loubmanifested. We also established a fascinating relationship between the face- centre-magic squares and the overall composite magic sums, a relationship so close in look and in application in this realm as the Einstein Equation .

In this work, concrete examples of the Minimal Bull Eye Composite Loub Magic Square Infinite Additive Abelian Groups are presented. The minimum Loub Magic Square is the and so the minimal Composite Loub Magic Square is the . We construct its Bull Eye and proved that the Bull Eyealso forms an Infinite Additive Abelian Group. Here, Bull Eye Loub Magic Squares are constructed as well as Bull Eye Composite.

We showcased that the set of the Minimal Bull Eye Compositeforms an infinite additive abelian group, an idea that does not require a deep mathematics to conclude its generalization.The whole concept is based on intimate survey of the structure of the centre piece and magic sum relationship formula, the mighty formula of magic squares realm. We will rightly-so introduce a new result on it.The minimum Loub Magic Square is the for Square is trivial for it is isomorphic to the underlined set of entries of the square so considered and Loub does not exist. Since the minimum is considered to be the , the minimum compositeis and its minimal Bull Eye is because an infix adjoinment of 17 times the face-centre magic squares is more than necessary, see (Babayo and Garba, 2015*l*).

**2.6Conclusion:**We could have reviewed some literature of interesting and revelant concepts of the Transformation Semigroup,Symmetry Group, Fibonacci Semigroups, Finite Fields, Rhotrix Theory and Magic Squares in general, but reviewingthem here will make this dissertation looks cumbersome, but consult (Babayo and Garba, 2015f) for our contribution to finite abelian groups and in other to see that the entries of the aforementioned squares can never be round/cyclic enough, see the Lindemann Theorem in (Babayo and Garba, 2015g).

**CHAPTER THREE**

SOME ALGEBRAIC STRUCTURES OVER THE LOUB MAGIC SQUARES

**3.1. Introduction**

In this chapter, we introduce the permutation and its composition over the Loub Magic Squares Semigroup to form a subgroup of the Symmetry Group which by analogy to the Fibonacci Group is termed the Tetrabonacci Group**.**If the multiset of natural number is the underlying set of arithmetic sequence arranged in the Loub Magic Squares, the set of the squares equipped with the binary operation of matrix addition forms a semigroup. permutation over the Loub Magic Squares enclosed with its composition forms the Symmetry Group of odd lengths (Babayo and Garba, 2015h). Not only to generate Symmetry Groups of odd lengths with the aforementioned squares is the aim of this work, but to minimize the number of the squares that generates it.

The job here, however along with caution of the Cayley Theorem, is to find the minimum number of the Loub Squares that will completely cover the structure of all the elements of the Transformation Semigroup of odd lengths map (Babayo and Garba, 2015i).

The Rhotrix Subelements of the squares are the rhomboidal array of number that are sitting in the magic squares, see also (Babayo and Garba, 2015c). The matrix subelements have analogous meaning and are considered babyish tautology, but we need the 2 arrays together to get the method: Rhotrix-Matrix Construction of the Loub Magic Squares. This implies that one of the newest realms of mathematics, the Rhotrix Theory has now gotten another genuine application. We showcased via concrete examples that the Rhotrix Subelements of the aforementioned squares form infinite additive abelian group.

**3.2.Tetrabonacci Subgroup of the Symmetry Group over the Magic Squares Semigroup**

The underlying sequencewe consider herein the aforementioned magic squares is the arithmetic sequence with unity common difference**.**We construct the subgroups of the Symmetry Groups of lengths 9 and 25. This does not however side-track generalization as well as the generalization of the general concepts explicated in this work as we finally set a conjecture**.**

In the permutation of the Symmetry Group, only the final effect is recognized; but for the construction of the aforementioned subgroup, both the motion which may be considered as clockwise + NE-W-S or anticlockwise + NW-E-S Loub Procedures of construction where NE stands for North-East, W stands for due West, S stands for due South, and NW and E stand for the analoguesand the effect which has to do with the final result of disjoint cycle elements of the permutation**.** This is of double interest for magic squares by themselves are energizing and important**.** See the work of (Sree, 2005) for some of such stimulations**.**

With the composition of the maps on the 4 elements per the general 8 miscellany effects (Gan *et al*., 2012) of rotations and/or reflections of 1 Loub Magic Square defined on the set of their permutations, the Tetrabonacci Groupan idea analogous to the Fibonacci Groupis formed.

**Definition 3.2.1.** A permutation is a bijective map from a set to itself**.**

The collection of all such maps with composition of maps forms the Symmetry Group, always of order n factorial () , and is called the Symmetry Group of length n**.** The n! order is obvious from the definition of a permutation map and a factorial function**.**

**Example I.** Consider the geometric construction of 2 per 24 elements of the Symmetry Group of length 4 and their composition as follows: Map the numbered (usually) vertices of the square to vertices of the same square as follows:

⟶and

i**.**e**.** in the mapping notation as and , and in the cyclic notation as and respectively**.** The composition \* of the two elements is

This confirmed that the 2 elements are inverses of each other with the identity, (1) standing for and for the map**:**

This map and its composition is imbibed and applied over the squares with sandwich of clockwise+ NE-W-S/ NW-E-S and anticlockwise+ NE-W-S/ NW-E-S Procedures**.**

**Definition 3.2.2.** The Fibonacci groups are defined by the presentation

where and all subscripts are reduced modulo n**.**

The generalization of the Fibonacci Groups (and their associated semigroups) are denoted and defined by the presentation

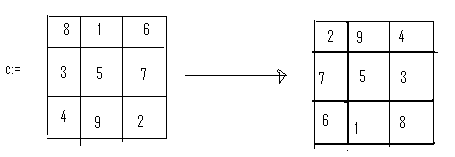
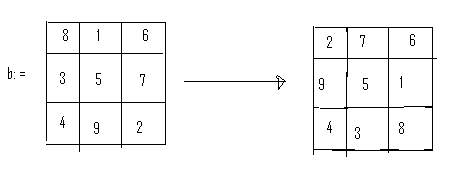
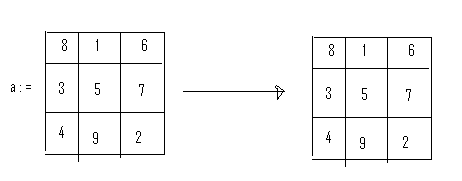
here and all subscripts are assumed to be reduced modulo n**.**We got this definition from (Umar, 2012) and (Colin *et al*., 2009). For presentations, refer to (Rukuc, 2000).

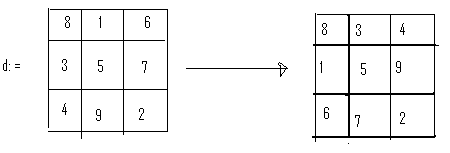
We construct the Tetrabonacci Subgroup of the Symmetry Groups of length 9 in a Loub Magic Square Semigroups and of length 25 in a Loub Magic Square Semigroups, and conjecture finally that this can be generalized**.**

The Tetrabonacci Subgroups under discussion are always of order 4, and are always having the identity element, which is a basic Loub Magic Square with the clockwise and NE-W-S Procedures; the next element is the rotation by 90 degrees followed by a reflection along the main column, the 3rd element is a rotation of the first by 180 degrees and the last element is a rotation of the second one by 180 degrees**.** This process will establish the Tetrabonacci Subgroups of the Symmetry Groups of any finite length as we propose**.**

**Examples II**

1. **The Tetrabonacci Subgroup of the Symmetry Group of Length 9 over the 3 Loub Magic Squares**





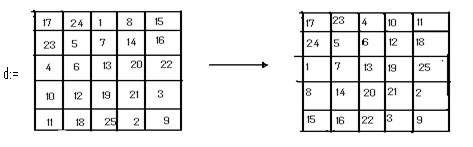
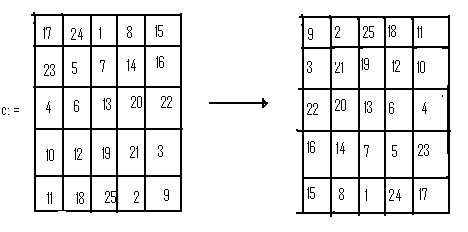
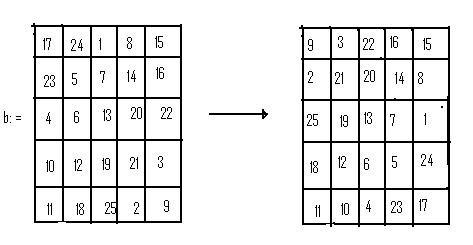
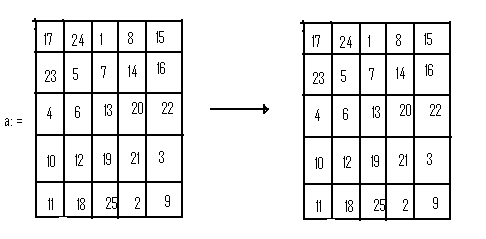
This maps 4 where 1 is a constant (an identity map) out of the 8 miscellany effects of rotations and/or reflections of a Loub Magic Square via sandwich**.**

This is the usual cyclic notations of the permutation notations widespread in introductory abstract algebra, see also (Joseph, 2005)**.**

It can be laconically visualized that where \* is a composition of map**.**

Thus, is a Tetrabonacci Subgroup of the Symmetry Group over the Loub Magic Squares Semigroup**.**

1. **The Tetrabonacci Subgroup of the Symmetry Group of Length 25 over the Loub Magic Squares**



,

,

, and

**.**

The set equipped with the composition of map forms the aforementioned group**.**

**Conjecture 3.2.3.** There exists a Tetrabonacci Subgroup of the Symmetry Group of any finite odd squared length over the Loub Magic Square**.**

The Loub Magic Squares equipped with the matrix binary operation of addition form a semigroup if the underlying set considered is the set of natural number**.** Loub Magic Square is considered trivial for it is isomorphic to the underlined set or multiset**.**For semigroup definition, see also (Howie, 2003).

**3.3.Symmetry Group of Odd Lengths over the Loub Magic Squares Semigroup of Multiset of Natural Number**

Given a Loub Magic Square, there are 7 miscellany effects formed as a result of rotations and/or reflections of it if the underlying set is the set of integer number, but if it is its multiset, 4 miscellany effects are recorded for 3 are missing as a consequence of isomorphism. If a permutation is defined on the collection of the squares and their effects, the collection of the maps and its composition forms the SymmetryGroup of odd length. The odd length is because the permutations are over the Loub Magic Squares and Loub Magic Squares are of the form where n is odd except , the triviality.

It will be remarkable if the number of Loub Magic Squares is minimized. To economize space, we showcased that this is true for SymmetryGroup of length 3 and no deep mathematics is required to make such a sudden conclusion of generality.

**Definition 3.3.1.** Let S be a non-empty set and M(S) be the collection of all maps from S to S. The collection of all invertible maps in M(S) is called a permutation.A permutation is a one to one map of a set onto itself.

**Theorem 3.3.2.** The set of all permutations of a non-empty set with respect to its composition is a group. This group is called the Symmetry Group of length n. Here, we restrict n to be odd.

Let us consider two regular transparent polygon glasses one on top of the other, with corners painted with colours a, b, c, ... ,n each, which makes it easy to distinguish between motions and effects. Repositioning the top polygon so that different colours match except the first colour matchings, including turning upside down of the top polygon, we get the elements of the permutations.

**Theorem 3.3.3.** The order of is n factorial ().

**Proof.** This follows from the definition of factorial function.

**Examples.**

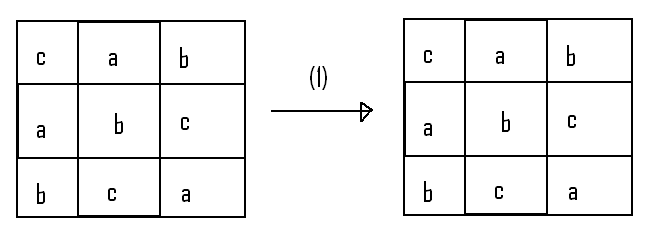
Loub Magic Square is trivial and Loub Magic Square does not exist. So we start with . For more details of the Symmetry Groups, see (Vasistha and Vasistha,2006).

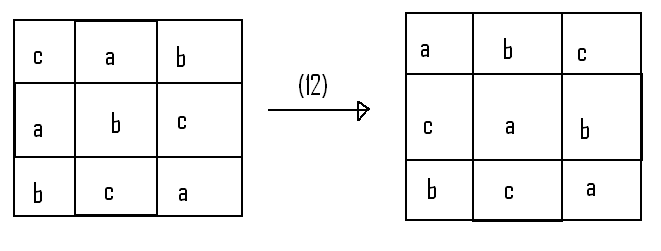
**3.3.4.Symmetry Groups over the Loub Magic Squares**

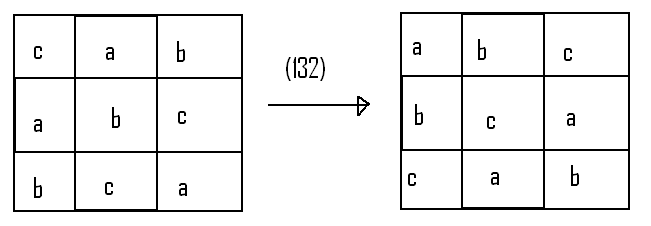
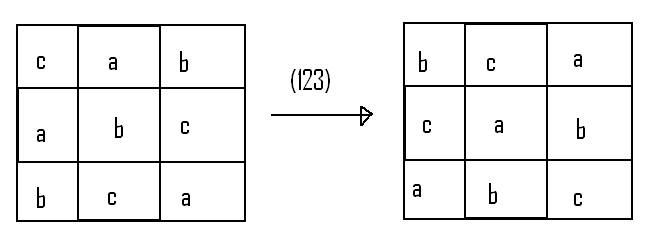
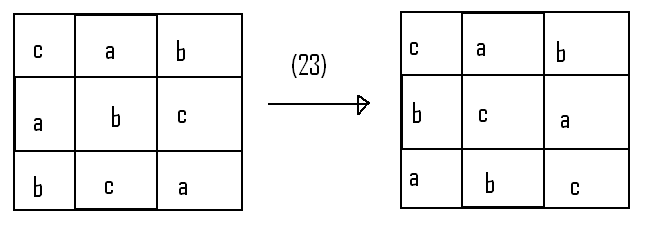
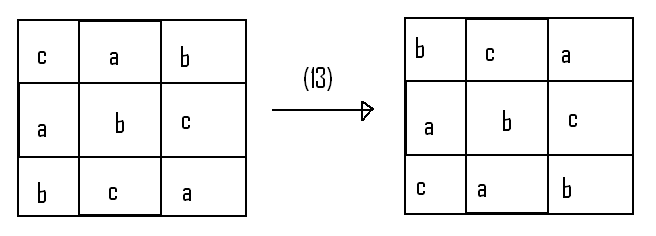
The multiset of natural number is considered because its Loub Magic Square is finer and have impler generalization, an element of the set of Semi Pandiagonal Loub Magic Squares, as in the above, unlike the set of natural number or of integer number. We tend to vividly outline that by swapping only rows or only columns of the generalized square, all the other possible effects will evolve. See the generalized squares over set of natural number in (Lee, 1986).

**3.3.5. Explanation.** Leave the square as it is, you have one element. Swap row1 with row 2, you have the 2nd element. Swap row 1 with row 3, you have the 3rd. Swap row 2 with row 3, you have the 4th. Swap row 1 with row 2 and row 2 with row 3, you have the 5th. And, swap row1 with row 3 and row 3 with row 2, you have the 6th element.

Check that any miscellany Loub Magic Square effect is isomorphic to one of the 6 below.

We now figure out the permutation as follows:





**3.3.6. Minimization.** We showcase that all the elements of the permutations above are in the two Loub Magic Squares of multisets:

:

andrespectively.

We now define a permutation on columns or rows of the above elements of the set of the Semi Pandiagonal Loub Magic Squares as follows:

andin permutation notations respectively, and in the cyclic notations:

respectively.

The collection of these elements with the composition of maps gives This result is true for all n as we presume.

**Remarks 3.3.7.** The later form of permutation is finer than the former. We move from symbols to number in the minimization to vividly intimate the clear nature of the elements of the permutations as they are usually represented with number rather than symbols.

**3.4.Transformation Semigroup of Odd Lengths over the Loub Magic Squares of Multiset of Natural Number**

If the Loub Magic Square subsets are able to construct both the permutation subsets of the collection of maps from a set to itself and both the maps form respective Symmetry Group (Joseph, 2005)and Transformation Semigroup(Rukuc, 2000) with respect to the composition of maps and the set of the Loub Magic Squares equipped or enclosed with matrix binary operation of addition is an abelian group, then the Cayley Theorem for both the group and the semigroup still stand while all the job joyously jugs for the subsets of the squares so considered are not groups or semigroups by themselves, see**(**Babayo and Garba, 2015i).

We know it may be exactly possible we can define elementary row and column operations on a Loub Magic Squares Set to construct the Transformation Semigroup, but never attempted it for the higher the number of entries in the set n of theTransformation Semigroup , the more cumbersome is such a set of all such maps that are apt to be equipped with composition to form it. The order of and the order of , and In this work, it is showcased that the Loub Magic Square mappings will generate maps to be equipped with composition to form the transformation semigroup. To economize space, we present a concrete example of the since Loub Magic Square is a triviality and does not exist. The n has to always be odd matching the exact definition of the Loub Magic Squares.

**Definition 3.4.1.** A multiset is a collection of objects in which repetition of elements is significant. See(Tella and Daniel, 2009).

Here, for their Loub Magic Squares are not isomorphic: We are on ordered multiset.

**Definition 3.4.2.** Full transformation semigroup is the semigroup of all maps from a set to itself equipped with composition of maps as an operation. For details, see (Rucuc, 2000).

If the set, then the assigned full transformation semigroup is of order 27.

The following is . The elements are in mapping notation.

**Definition 3.4.3.** The rank of . It is the cardinality of a minimal generating set. We found the above definition in (Howie, 2003).

has a lower rank than the aforementioned rank of the squares as generates it, but we lost intimation of the nature of its elements.

**Theorem 3.4.4. (Cayley Theorem Analogue for Semigroups).** Every semigroup is embeddable in the transformation semigroup.

For details, see (Howie, 2003). This theorem is referred to as Preston-Wagner Theorem.This theorem came later than the present definition of semigroup.

**Definition 3.4.5.** Let S be a non-empty set and M(S) be the collection of all maps from S to S. The collection of all invertible maps in M(S) is called a permutation. Apermutation is a one to one map of a set onto itself.

**Theorem 3.4.6.** The set of all permutations of a non-empty set with respect to its composition is a group. This group is called the Symmetry Group of length n. Here, we restrict n to be odd.

**Theorem 3.4.7.** The order of is n factorial ().

**Proof.** This follows from the definition of factorial function.

**Theorem 3.4.8 (Cayley Theorem for Groups).** Every finite group is embeddable in the Symmetric Group.

For details, see (Joseph, 2005). This theorem came earlier than the present definition of group.

We showcase that all the elements of the permutations above are in the two Loub Magic Squares of multisets: and :

andrespectively.

We now define a permutation on columns or rows of the above elements of the set of the Semi Pandiagonal Loub Magic Squares as follows:

andin permutation notations respectively, and in the cyclic notations as: respectively.

The collection of these elements equipped with the composition of maps gives This result is true for all n as a presumption.

**3.4.9.Transformation Semigroup of Odd Lengths over the Loub Magic Squares of Multiset of Natural Number**

The rank of the Transformation Semigroup is the number of elements generating it. In this session, it is explicated that the rank of the Loub Magic Squares of the full transformation semigroup is 7. The underlying set so considered of the sequence arranged in the square is a multiset of natural number.

Without the repetitions, the possible arrangement of the multisets of choosing 3 elements at a time are

Include [[1, 2, 3]] in the enumeration. Each of the above multisets represents quite a number of miscellanies. [[1,2,3]] represents miscellany sets, [[2,2,1]] represents miscellany sets: .

The collection of all the Loub Magic Squares of the enumerated elements generates as follows:

[[1, 2, 3]].

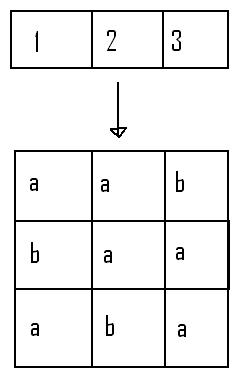
[[1, 1, 2]].

[[1, 1,3]].

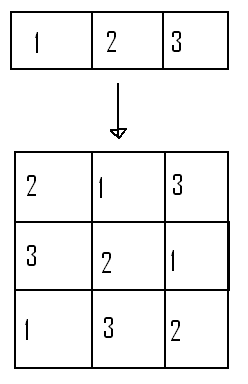
[[2, 2, 1]].

[[3,3,1]].

Without repetitions, the mappings of the following giveall the elements of the map in



**Example.** The possible maps in

 are

,

The rank of Loub Magic Squares of the Full Transformation Semigroup is 7.

**3.5.Rhotrix Subelement Subgroups of the Loub Magic Squares Infinite Additive Abelian Group**

The aims of this session are two: First, to present the Rhotrix-Matrix Construction of and Magic Squares where the is Loub The construction of Magic Squares via Rhotrix-Matrix Method is one of the finest achievement of one of the newest realm of mathematics, the Rhotrix Theory first introduced by (Ajibade, 2003) which has to do with the rhomboidal array of number.

A rhotrix is a rhomboidal array of number and a matrix is a rectangular array of number. Usually, the rhotrix less the matrix of equal dimension with a multiple of 3 number of elements, and all rhotrices are of odd dimensions. So far, the applications of rhotrices, unlike matrices, are seemingly very rare or usefully verbatim virtuoso of matrices.

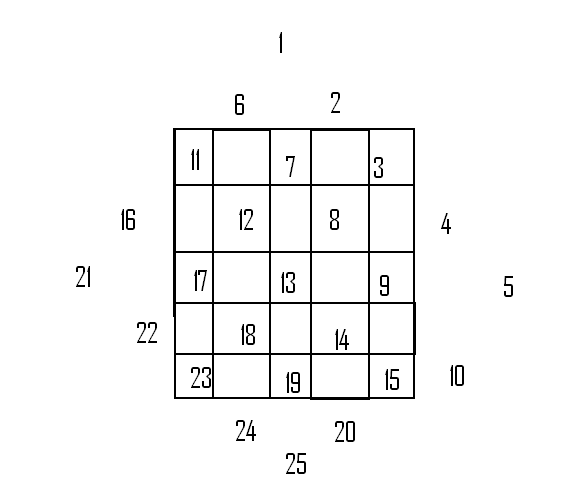
We firstly so set to provide the construction of for the is isomorphic to any chosen underlined set. We now present the construction of We stop to economize space.

**3.5.1. The Rhotrix- Matrix Construction of Magic Square**

Consider an arbitrary arithmetic sequence: Write the sequence into a square matrix

Transform the matrix to a rhotrix as follows:

Draw a Subelement Draught on the rhotrix where the heart of the rhotrix is the centre piece of the Draught.



Fill the eastern empty cells with the triangular array of number outside the draught on the west of the heart (centre piece), the southern empty cells with number on the north, the western with the number on the east and the northern with the number on the south.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 11 | 24 | 7 | 20 | 3 |
| 4 | 12 | 25 | 8 | 16 |
| 17 | 5 | 13 | 21 | 9 |
| 10 | 18 | 1 | 14 | 22 |
| 23 | 6 | 9 | 2 | 5 |

**3.5.2.Magic Rhotrix Subelement Subgroups of the Loub Magic Squares Abelian Group**

We know that the set of Loub Magic Squares over the set of natural number equipped with the matrix binary operation of addition forms a semigroup, and over the set of integer number forms a group-enclosed with the same operation.

Consider arbitrarily the following three general Loub Magic Rhotrices:

We now show that they satisfy **theAxioms of a Group** as follows:

1. **Closure.** Let
2. **Associativity.** Since we are dealing with integer number as entries of the rhotrices, then

We show the two miscellany **axioms** as follows:

1. Since we are dealing with the integer number, then , a singleton set with the arithmetic sequence 0, 0+0, ..., 0+0+0+0+0+0+0+0+0 can be arranged in the square to form an identity element as follows:

Then it is obvious that if a is a Loub Magic Rhotrix, then .

1. Let arbitrarily the following Magic Rhotrix be LoubThen is such that

Thus, they are **inverses** of each other.

1. **Commutativity.**

These are the Rhotrix Subelement **Subgroups** of the Loub Magic Squares Infinite Additive Abelian **Group**.

**Remarks 3.6.** Magic matrices are trivial by the widespread definition of magic squares. Analogous to the above, the magic matrices (seems tautology by the contention) form an infinite additive abelian group with respect to matrix addition operation.

Because Rhotrix Theory, Transformation Semigroup(the single most important semigroup), Symmetry Group (the single most important group), Fibonacci Semigroup, Multiset Theory, Semigroup Theory, Group Theory, Ring Theory, Field Theory and Matrix Theory are all in this mathematical realm; this realm fit the definition of “Good Mathematics” well.

**3.7.About Methodology**

This entire chapter is devoted to methodology. That is the methods and the study of the methods employed for the research. As earlier lamented, the most important method use to carry out this research is ‘perception’ inherited from the ancient times. Be that as it may, according to some experts in the realm of ‘pure mathematics’, there is no room for methodology. The methods inherited from the ancient times are enough and are as follows: Methods of Proof by Contradiction, Methods of Proof by Counter Example, Method of Proof by Induction Hypothesis and Method of Proof using Propositional Logic. We used the Method of Proof by Counter Example. This method is use to disprove a universal statement. Though we did not use the Method of Proof by Induction Hypothesis directly, yet we used it indirectly from the fact that we do not go to degree and concluded that we prove mathematics theorem which is a syndrome that affects some parts of developing realms of mathematics. It is because of this method that it is decided not to include 0 in the set of natural numbers. We also use the Method of Proof using Propositional Logic in the general Lou Magic Square.

All the concepts (including many definitions and theorems) in this dissertationare original. We used SVOOCA System of Sentencing, SOSCA Formula of Adjectives, Anglo Saxon Words, and Periodic Sentences for the demand of clarity. This work is original, elegant and easy to comprehend for we balance, virtually all, the concepts presented in this thesis with explicit language. We also use title style of capitalization in the not proper noun clauses and phrases. Chidume(2006) said you can find two mathematicians who claim to be functional analysts but they do not understand each other. Such happens in the realms that are no more underdeveloped.

**CHAPTER FOUR**

CONSTRUCTION OF SEMIGROUPS, GROUPS, SEMIRINGS AND FIELDS USING THE SET OF LOUB MAGIC SQUARES

**4.1.Introduction**

In this chapter, we present the Loub Magic Squares over multiset of rational number of the form infinite algebraic field, where denotes the multiset of integer number. By the Loub Magic Squares L (as we denote it), we are to understand the set of magic squares formed by the De La Loub Procedure. It is explicated that the set equipped with the binary operation of matrix addition (as we denote it) forms an infinite additive abelian group, and the set enclosed with the rational number multiplication (as we denote it) forms an infinite multiplicative abelian group if the underlying set so considered of the entries of the aforementioned squares is the multiset of the aforementioned set of number. Then forms an infinite field (Babayo and Garba,2015a).

In this chapter, mystic miscellaneous algebraic properties of the set of Composite (Nested) Loub Magic Squares are vividly visualized. And, verbatim virtuoso of algebraic properties of the Loub Magic Squares viz: Eigen Group, Magic Sum Group and Centre Pieces Group eloquently viewed the algebraic properties of its Composite. It is also showcased that both the 2 sets equipped with the binary operation of matrix addition form infinite additive abelian groups (Babayo and Garba, 2015e).

By Type I Composite of Type I Loub Magic Squares, we understand the set of all Type I Loub Magic Squares such that each cell (grid) of its element is a Type I Loub Magic Square, the magic squares formed by the De La Loub Procedure over the arithmetic sequence of common difference of 1. In this pioneering work, it is explicated that the aforementioned set equipped with the binary operation of matrix addition(as we denote it) forms an infinite additive abelian group. We also showcased that the set of Type I Composite of Type II Loub Magic Squares, the magic squares constructed with the repeating-pattern-sequence Loub Procedure, equipped with the aforementioned operation and with the rational number multiplication (as we denote it) forms an Infinite Field if the underlying multiset of entries of the squares is of the rational number of the form. forms an infinite field (Babayo and Garba, 2015k).

In this chapter also, concrete examples of the Minimal Bull Eye Composite Loub Magic Square Infinite Additive Abelian Groups are presented. The minimum Loub Magic Square is the and so the minimal Composite Loub Magic Square is the . We construct its Bull Eye and proved that the Bull Eyealso forms an Infinite Additive Abelian Group.

**4.2. The Loub Magic Squares over Sets ()Semigroup and Group**

We present that the set of Loub Magic Squares over the set of natural number equipped with the binary operation of matrix addition forms a semigroup, and over the set of integer number forms a group-enclosed with the same operation.

**Definition 4.2.1.** The square grid of cells is said to be Loub Magic Square if the following conditions are satisfied.

1. are on the same main column or row and are on the same main column or row,

where is the greatest integer less or equal to x, T is the transpose (of the square),

k is the magic sum (magic product is defined analogously) usually expressed as from the sum of arithmetic sequence, where j is the common difference along the main column or row and a is the first term of the sequence and .

We use Definition 4.2.1 to prove Theorem 4.2.2that follows:

**Theorem 4.2.2.** forms an infinite commutative semigroup if the underlying multiset is of natural number and it forms an infinite additive abelian group if the underlying multiset is of integer number.

**Proof.**

1. **Closure Property.** Let . Then, by Definition 4.2.1, ,

are on the same main column or row; are on the same main column or row, ,,

are on the same main column or row and are on the same main column or row.

Then,

, are on the same main column or row;are on the same main column or row since are on the same main column or row, are on the same main column or row , are on the same main column or row and are on the same main column or row.

ii. **Associativity Property.** Let . Then, by Definition 4.2.1, ,

are on the same main column or row, are on the same main column or row,, ,

are on the same main column or row , are on the same main column or row, , and

are on the same main column or row and are on the same main column or row.

Then,

, and are on the same main column or row and, then are on the same main column or row and are on the same main column or row , and are on the same main column or row and are on the same main column or row, and are on the same main column or row and are on the same main column or row.

We pose here if the underlying set of entries of the square is a multiset of natural number.

iii.**Identity Element Property**. if the following conditions are satisfied.

are on the same main column or row and are on the same main column or row, whence

1. **Inverse Element Property.** Given that,

and

are on the same main column or row and are on the same main column or row, there exists such that ,

, and

are on the same main column or row and are on the same main column or row.

Thus, **.**

1. **Commutativity Property.**

and

are on the same main column or row as well as are on the same main column or row , .

**4.3. Minimum Zero-Centre-Pandiagonal Composite Type II (a) Loub Magic Squares over Multiset of Natural Number as a Semiring**

In this session, weexplicate that the set of the Minimum Zero-Centre-Pandiagonal Composite Type II (a) Loub Magic Squares over Multiset of IntegerNumber (as we denote it) forms an additive abelian group if equipped with the binary operation of matrix addition (as we denote it) and if it is enclosed with the operation of integer multiplication (as we denote it), it forms a multiplicative semigroup with identity. That is, forms a semiring for it satisfies the other two axioms of a semiring (Babayo and Khan, 2015).

By Zero-Centre-Pandiagonal Composite Type II (a) Loub Magic Squares over Multiset of Natural Number, we understand the set of magic squares formed by the De La Loub Procedure of repeating-pattern-sequence such that each of its cell is a Loub Magic Square of Type II(a) and such that the diagonal magic squares across the face-centre magic square contain only zero entries.

We showcased that the Minimum Zero-Centre-Pandiagonal Composite Type II (a) Loub Magic Squares over Multiset of Natural Number (as we denote it) forms an additive abelian group if equipped with the matrix binary operation of addition (as we denote it) and if it is enclosed with the natural number operation of multiplication (as we denote it), it forms a multiplicative semigroup with identity.

Arbitrarily, Minimum Zero-Centre-Pandiagonal Composite Type II (a) Loub Magic Squares over Multiset of Natural Number is considered not squares such that to economize space, and not , the triviality, for it is isomorphic to the aforementioned underlying multiset.

We also consider the sequence … n times rather thana, though the later will also give an analogous result; but presenting both the two is a babyish tautology.

By the Null Magic Sum and Magic Product Bull Eye Loub Magic Square, we understand the magic squares constructed with the De La Loub Procedure incorporated with an infix adjoinment of 17 times the centre piece of the square such that its magic sum as well as its magic product is zero. In this session, we showcase that the set NBL (as we denote it) of the aforementioned magic squares equipped with both the matrix binary operation of addition (as we denote it) and the integer number binary operation of multiplication (as we denote it) forms a ring (NBL,,) as the consequence of the semiring theorems.

We consider a Minimum Bull Eye Loub Magic Squares, the Squares, having entries the arithmetic sequence with 17 times repetition of the centre piece to economize space, noting that Loub Magic Square is a triviality for it is isomorphic to the underlying multiset and Loub Magic Square does not exist. Be that as it may, the Minimum Bull Eye Loub Magic Square is . We explicate that the set of the Bull Eye Loub Magic Squares having Null Magic Sum and Magic Product satisfy the properties of an algebraic ring as a corollary. The proof of the Loub Magic Squares having Null Magic Sum and Semi Null Magic Product over Multiset of Integer Number is its consequence and a corollary of the consequence of this consequence is that the Loub Magic Squares having Null Magic Sum and Semi Null Magic Product over set of Integer Number form a ring.

**4.3.1.Semiring**

A semiring is an algebraic structure, consisting of a nonempty set R on which we have two operations, addition and multiplication such that the following conditions hold:

1. Addition is associative and commutative and has a neutral element. That is to say, and for all and there exists a special element of R, usually denoted by 0, such that for all . It is very easy to prove that this element is unique.
2. Multiplication is associative and has a neutral element. That is to say, for all and there exists a special element of R, usually denoted by 1, such that for all . It is very easy to prove that this element too is unique. In order to avoid trivial cases, we will always assume that, thus insuring that R has at least two distinct elements.
3. Multiplication distributes over addition from either side. That is to say,

and for all

1. The neutral element with respect to addition is multiplicatively absorbing. That is to say, for all

The above definition is available in **(**Jonathan, 2008).

**Definition 4.3.2.** A Semi Null Magic Product is a Zero Valued Product over the main row, the main column and the two diagonals of a magic square.

**4.3.3. Ring**

A non-empty set R together with two binary operations called addition and multiplication denoted by and respectively is called a Ring, if the following postulates are satisfied.

1. is an abelian group.
2. Multiplication is associative, i.e.
3. Multiplication is distributive with respect to addition, i.e. (Left Distributive Law) and (Right Distributive Law).

The above definition is available in (Sreerangini and Madhukar, 2012a, b).

**4.3.4.Minimum Zero-Centre-Pandiagonal Composite Type II (a) Loub Magic Squares over Multiset of Natural Number as a Semiring**

Let the set of the Minimum Zero-Centre-Pandiagonal Composite Type II (a) Loub Magic Squares over Multiset of IntegerNumber be denoted by , let the binary operation of matrix addition be denoted by , and let the integernumber operation of multiplication be denoted by . Then, we present the following theorems:

**Theorem 4.3.5.** forms an additive abelian group.

**Proof.** Arbitrarily considering(a denotation of the aforementioned squares of ) we define as follows: Let where . Then:

1. is **closed** with respect to : From the above definition, if , then . This is more vivid by letting (say) .
2. is **associative**: For

whence where .

1. The **identity element** 0 is .
2. Given , there exists an **inverse** of

it, Y expressed as such that .

1. is **commutative**: whence the matrix binary operation of addition over set of integer number is commutative (inheritance).

Thus, forms an additive **abelian group**.

**Theorem 4.3.6.** forms a multiplicative semigroup with identity.

**Proof.** Let . We define

.

Then

1. is **closed** with respect to For from the above definition. This is vivid by intimate look at the pattern of elements in F.
2. **Associativity**: is associative for because .
3. The **identity element** is **.**

Thus, forms a multiplicative**semigroup** with identity.

**Remark 4.3.7.** For all and

The neutral element with respect to addition is multiplicatively absorbing. That is to say, for all whence

**Theorem 4.3.8.** forms a semiring.

**Proof.** This follows immediately from Theorem 4.3.5, Theorem 4.3.6and Remark 4.3.7.

**Remarks 4.3.9.**Every Minimum Zero-Centre-Pandiagonal Composite Type II (a) Loub Magic Squares under discussion in this work has about 4 miscellany effects of rotations and/or reflections. Considering 1 out of the 4 effects is arbitrary. Although the Magic Squares presented here are not the basic (obvious) ones, yet they are Pandiagonal Loub with unique magic sums and products.

**4.3.10. Corollaries of Theorem 4.3.9**

Let NBL denotes the set of the Null Magic Sum and Magic Product Bull Eye Loub Magic Squares, let NCL denotes the Null Magic Sum and Semi Null Magic Product Composite Loub Magic Squares; and NL denotes the Null Magic Sum and Semi Null Magic Product Loub Magic Squares. Then:

**Corollary 4.3.11.** (NBL,) is an infinite abelian group.

**Proof.** Arbitrarily considering we define as follows: Let where

. Then:

=

i. is **closed** with respect to : From the above definition, if , then

.

ii. is **associative**: For

whence where .

iii. The **identity element** 0 is .

iv.Given that , there exists an **inverse** of it, Y given by such that .

v.is **commutative**: whence the matrix binary operation of addition over set of integer number is commutative (inheritance).

Thus, forms an additive **abelian group**.

**Corollary 4.3.12.** forms a multiplicative semigroup without identity.

**Proof.** We define

. Then:

is **closed** with respect to For from the above definition. This is vivid by noting that we hereby intimate magic product (not magic sum).

1. **Associativity**: is associative for because .

Thus, forms a multiplicative **semigroup** without identity.

**Corrollary 4.3.13.**is both left and right distributive over defined on NBL, i.e. and

**Proof.**

**Theorem 3.4.14.** (NBL,) is a ring with no identity.

**Proof.** This follows immediately from Corollaries 4.3.11, 4.3.12 and 4.3.13.

**Corollary 4.3.15.** (NCL,) is a ring with no identity.

**Proof.**We define

.

Then:

We define

The result follows immediately from Corollaries 4.3.11, 4.3.12, 4.3.13.

**Corollary 4.3.16.** (NL,) is a ring with no identity.

**Proof.**

We define. Then:

We define

The result follows immediately from Corollary 4.3.15.

**Conjecture 4.3.17.** The Loub Magic Squares over Multiset of Integer Number (Type II) forms a ring.

Examples of Loub Magic Square over Multiset of Integer Number are

... as follows:

,

.

**4.4.Loub Magic Squares over Multiset of Rational Number as an Infinite Field**

The Loub Magic Squares L equipped with the binary operation of matrix addition forms a semigroup if the underlying set considered is the multiset of natural number. If we underline the multiset of integer number as entries of the square, forms an infinite additive abelian group.

The Loub Magic Squares over the multiset of rational number of the form forms an infinite multiplicative abelian group,, thus, making an infinite field. This is not the explication of the definition of the field presented in (Sreeranjini and Madhukar, 2012a), but the two are analogous.

Arbitrarily, Semi Pandiagonal Loub Magic Squares is considered not squares such that to economize space, and not such a square to dodge near bias choice.

The multiset of rational number of the form is aptly considered even though the multiset of rational number of the form, a special type of scalar multiplication, will do. Also, we consider the sequence than a, b, c, ... n times, a, b, c, ... n times, ...n times though the later will also give an analogous result; but presenting both the two is a babyish tautology.

**Definition 4.4.1.** If is an additive abelian group and is a multiplicative abelian group, then is a field.

**4.4.2. The Loub Magic Square Fields**

The underlying multiset of the infinite additive Loub Magic Squares abelian group is and the underlying multiset of the infinite multiplicative Loub Magic Squares abelian group is. The whole concept is based on the manifestation of Loub Magic Squares where n is odd having both the magic sum and the magic product.

a is the multiset considered, where is the set of positive integers (greater than or equal to 1) for is a triviality and the oddest (even) prime, does not exist.

The corresponding set of magic squares of entries the sequence of elements of the multiset above are ...**.**

For example,

and.

.

**Theorem 4.4.3.** is an additive abelian group.

**Proof.** Arbitrarily considering, we define as follows: Let where

Then:

1. is **closed** with respect to : From the above definition, if , then . This is more vivid by letting (say) .
2. is **associative**: For whence

where

1. The additive **identity** element I is where each 0 except the first entry of the square is a series of 0s correspondingly n times.
2. Each element of has an **inverse**: If , then where is the square with magic sumformed as a result of scalar multiplication of entries in D having magic sum M(S) by .

For example, the inverse of is

1. is **commutative**: whence the matrix binary operation of addition over set of integer number is commutative (inheritance).

Thus, is an infinite additive **abelian group**.

**Theorem 4.4.4.** is an infinite multiplicative abelian group.

**Proof.** Let . We define . Then

is **closed** with respect to For from the above definition. This is vivid by intimate look at the pattern of elements in F: ie, jf, kg, lh, and mi.

**Associativity**: is associative for because .

The **identity** element I is where each is a product of s correspondingly n times.

Each element of has an **inverse**: If then

such that .

is **commutative**: since the entries in the square are rational number and rational number multiplication is commutative, we are done.

Thus, is an infinite multiplicative **abelian group**.

**Theorem 4.4.5.** is an infinite field.

**Proof.** This follows immediately from Theorem 4.4.3 and Theorem 4.4.4.

**4.4.6. Conclusion**

Every Loub Magic Square under discussion in this session has about 4 miscellany effects of rotations and/or reflections. Considering 1 out of the 4 effects is arbitrary. Although the Loub Magic Squares presented here are not the basic (obvious) ones, yet they are semi pandiagonal Loub with unique magic sums and products.

**Composite Loub Magic Squares Infinite Abelian Group as a Miscellany Case of the Loub Magic Squares Infinite Abelian Group**

It is remarkable that almost joyously the sets of eigen values, centre pieces and magic sums of the Loub Magic Squares Infinite Abelian Group form Infinite Additive Abelian Groups.

We highlighted consortium of miscellany effects of rotations and/or reflections and/or enumerations of the Loub Magic Squares to figure out the consortium of the Composites, see (Babayo and Garba, 2015j).

Establishing such a fact relationships set us conjecture that the Composite (Ahmed, 2004)Loub Magic Squares Infinite Additive Abelian Group is a miscellany case of the Loub Magic Squares Infinite Abelian Group.

**Definition 4.5.1.** A Composite Loub Magic Square is a magic square such that each of its cell (grid) is a Loub Magic Square. See also (Ahmed, 2004).

**Definition 4.5.2.** A Loub Magic Square of type I is a magic square of arithmetic sequence entries such that the entries along the main column or row have a common difference and the main column or row is the central column or central row respectively.

**Definition 4.5.3.** Loub Magic Squares of type II are magic squares constructed with Loub Procedure with repeating-pattern-sequence.

**Definition 4.5.4.** A least subelement of Loub Magic Square is a Square formed by removing boarder cells of the Loub Magic Squares.

**Remark 4.5.5.** The least subelement squares of Loub Magic Squares are subsets of the semi pancolumn magic squares and the least subelement magic square of the Composite Loub Magic Square is a Loub Magic Square.

**4.5.6.Centre Pieces and Magic Sums Abelian Groups**

**4.5.6.1. Centre Pieces Abelian Group**

The set of the centre pieces { of Loub Magic Squares equipped with integer addition forms an infinite additive abelian group. Given the centre pieces of Loub Magic Squares with corresponding formula:

,

1. is the centre piece of the Loub Magic Square with first term as and common difference along the main column as. Hence, the set is **closed**.
2. **Associativity.** This is an inherited property of the set of integer number:
3. The **identity element** is the zero centre piece, e.g. .
4. Given an arbitrary centre piece of the Loub Magic Square, there exists another centre piece of another Loub Magic Square having first term as and common difference along the main column or row as , thus its formulae is and is such that

, the identity centre piece.

are **inverses** of each other.

1. **Commutativity.**

Clearly,

The set equipped with the operation is an **abelian group**.

**4.5.6.2. Magic Sum Abelian Groups**

The set of the magic sums of Loub Magic Squares equipped with integer addition forms an infinite additive abelian group.

Given the magic sums of Loub Magic Squares with corresponding formula:

hen (as in the above):

Loub Magic Square with first term and common difference along the main column as

The axioms: follow, by analogy to the centre piece infinite additive abelian group established result, immediately.

**4.5.6.3.Eigen Values Abelian Group**

The eigen values computation in the magic squares is what is zealotly prophesized that magic squares are special type of matrices, hence the definition of the magic squares, we do not love to like such a sudden conclusion if loving liking forces choosing the definitions in terms of just the square grids (or cells).

We want to show through concrete examples that the set of Eigen Values of the Loub Magic Squares with the usual binary operation of integer addition forms a group.

We now conclude this session by showing that the set of eigen values satisfies **The Properties of an Additive Abelian Group** as follows:

**Closure Property.** Consider any 3 arbitrary Loub Magic Squares a, b, c; such that ; then as previously, the corresponding eigen values of a; the corresponding eigen values of b; are such that where are the corresponding eigen values of c.

**Associativity Property.** Since the set of Loub Magic Squares is a semigroup (which is easy to observe), the eigen values are associative.

**Identity Element Property.** The eigen value 0 is the identity element that corresponds to the sum of the Loub Magic Squares of opposite eigen values as in the above.

**Inverse Elements Property.** For any arbitrary eigen value corresponding to a Loub Magic Square m, there exists a eigen value corresponding to another Loub Magic Square such that equals the identity element which is formed as a result of matrix addition of the aforementioned Loub Magic Squares.

**Commutativity Property.** Integer number binary operation of addition is commutative.

This completes the proof. The idea of eigen values computation of a magic square is conceived from the work of (Babayo and Garba, 2015d).

**4.5.6.4. Composite Loub Magic Squares Infinite Abelian Group as a Miscellany Case of the Loub Magic Squares Infinite Abelian Group**

Letn stands for number of columns, d stands for common difference of entries and f stands for first term of the aforementioned square. Then denotes the sequences of Loub Magic Squares of type I of d common difference of its entries and of first terms f, denotes the sequences of Loub Magic Squares of type II(a) of d common difference of its entries and of first terms f, denotes the sequence of type II(b), denotes the sequence of the composites of , denotes the sequence of composites of , denotes the sequence of the composites of having entries or simply T(f), and denotes the sequence of the composites of having entries Then, the sequences are as follows:

**Remarks 4.5.6.5.** are enumerated analogously and ,, and .

**Theorem 4.5.6.6.** The set of ,, form infinite additive abelian groups.

**Proof.** The sum of two sequences of the types ,, is a sequence of their type. Thus, **Closure Property** is exhibited.

**Associativity Property.** This is an inherited property of closure above whence we have integer number entries in the sequences.

**Identity Property.** The identities are the sequences ,, respectively.

**Inverse Property.** Each element in ,, has an inverse. For example ,, have inverses ,, respectively.

**Commutativity Property.** Integer number binary operation of addition is commutative.

This completes the proof. An alternative proof may be by the fact that equivalence classes forms a free group.

**Remark4.5.6.7.** The 3 Eigen Values, Magic Sums and Centre Pieces of the Composite Loub Magic Square are 3 times that of the Loub Magic Square.

**Proof.** This is manifested clearly in the enumeration of ,, .

**Theorem 4.5.6.8.** The 3 Eigen Values, Magic Sums and Centre Pieces of the Composite Loub Magic Square that are multiples of that of the Loub Magic Square form Infinite Additive Abelian Groups.

**Proof.** If a set of integer number equipped with an operation is a group, then 3 times the set of corresponding elements of the set equipped with the same operation is also a group. This follows immediately from Remark 4.5.6.7.

**4.6.Type I Composite of Type I Loub Magic Squares as an Infinite Additive Abelian Group and of Type II as an Infinite Field**

The set of Loub Magic Squares of Type I equipped with the binary operation of matrix addition forms an infinite additive abelian group if the underlying set is of the multiset of integer number, and the set of Loub Magic Squares of type II equipped with the binary operation of matrix addition and with the rational number multiplication forms an infinite field if the underlying multiset is of integer number including some rational number of the form .

It is remarkable if the composite of the aforementioned types of magic squares form the aforementioned respective groups and fields. This is our proposed construction, and however along with the scope, a simple definition, a finer procedure and a newest generation of the Loub- manifested. We also established a fascinating relationship between the face- centre-magic squares and the overall composite magic sums, a relationship so close in look and in application in this realm as the Einstein Equation .

**4.6.1.Type I Composite of Type I Loub Magic Squares as an Infinite Additive Abelian Group**

**Proposition 4.6.2.** Let denotes the face-centre Type I Loub Magic Squares of the Type I Composite Loub Magic Squares . Then the Magic Sum of is n times the magic sum of .

**Proof .** This follows from a thorough check of the above sequences.

**Definition 4.6.3.** A Composite Loub Magic Square of Type I is the square matrix such that and

where runs over the natural number arithmetic sequence of common difference n of 1 to values of k and is the greatest integer less or equal to x.

**Theorem 4.6.4.** forms an Infinite Additive Abelian Group.

**Proof.**

1. We check **closure property** as follows:

nd

Thus, .

1. We show **associativity property** as follows:

,and

Thus, .

,

0 is the **identity element** having entries(except the first) series of zeros.

and

1. **Commutativity.**Since then

and

Thus, and

forms an Infinite Additive **Abelian Group**.

**Theorem 4.6.5.** The centre pieces and magic sums of the composite Loub magic of the square are given by respectively.

**Proof.** This is a mere averaging of 2 non-composite centre pieces and magic sums of Loub Magic Squares.

**Theorem 4.6.6.** The set of the centre pieces and the magic sums form respective infinite additive abelian groups.

**Proof.** This follows from the centre pieces and the magic sum infinite additive abelian groups of non- composite squares proof.

**4.6.7.Type II Composite of Type II Loub Magic Squares as an Infinite Field**

The underlying multiset of the Infinite Additive Composite Loub Magic Squares Abelian Group is and the underlying multiset of the Infinite Multiplicative Composite Loub Magic Squares Abelian Group is. The whole concept is based on the manifestation of Composite Loub Magic Squares where n is odd having both magic sum and magic product.

is the multiset considered, where is the set of positive integers (greater than or equal to 1) for is a triviality and the oddest (even) prime, does not exist.

The corresponding set of composite magic squares of entries the sequence of elements of the multiset above are ...**.**

,

.

**Theorem 4.6.8.** is an additive abelian group.

**Proof.** Arbitrarily considering, we define as follows: Let where

Then:

i.is **closed** with respect to : From the above definition, if , then . This is more vivid by letting (say)

…**.**

is **associative**: For

whencewhere.

1. The additive **identity** element I is where each 0 except the first entry of the square is a series of 0s correspondingly times.

iv.Each element of has an **inverse**: If , then where –D is the composite square with magic sum –M(S) formed as a result of scalar multiplication of entries in D having magic sum M(S) by .

For example, the inverse of is .

1. is **commutative**: whence the matrix binary operation of addition over set of integer number is commutative (inheritance).

Thus, is an infinite additive **abelian group**.

**Theorem 4.6.9.** is an infinite multiplicative abelian group.

**Proof.** Let . We define ⊗ as follows: . Then

is **closed** with respect to For from the above definition. This is vivid by intimate look at the pattern of elements in F.

1. **Associativity**: is associative for because .
2. The **identity** element I is where each

is a product of s correspondingly times.

1. Each element of has an **inverse**:

If  
 then

such that .

1. is **commutative**: since the entries in the square are rational number and rational number multiplication is commutative, we are done.

Thus, is an infinite multiplicative **abelian group**.

**Theorem 4.6.10.** is an infinite field.

**Proof.** This follows immediately from Theorem 4.6.8 and Theorem 4.6.9.

For the definitions of Group and Field, see (Sreeranjini and Madhukar, 2012a,b) and for the definition of Composite Magic Square, see (Ahmed, 2004).

**4.7.Concrete Examples of the Minimal Bull Eye Composite Loub Magic Square Infinite Additive Abelian Groups**

In this pioneering work, Bull Eye Loub Magic Squares are constructed as well as Bull Eye Composite.

We showcased that the set of the Minimal Bull Eye Compositeforms an infinite additive abelian group, an idea that does not require a deep mathematics to conclude its generalization.

The whole concept is based on intimate survey of the structure of the centre piece formula, the mighty formula of magic squares realm. We will rightly so introduce a new result on it.

The minimum Loub Magic Square is the for Square is trivial for it is isomorphic to the underlined set of entries of the square so considered and Loub - does not exist. Since the minimum is considered to be the , the minimum Composite- is and its minimal Bull Eye is because an infix adjoinment of 17 times the face-centre magic squares is more than necessary(Babayo and Garba, 2015*l*).

**Definition 4.7.1.** A Bull Eye Loub Magic Square is a magic square constructed by an infix adjoinment (in a special way) of the centre piece entry or its Loub Magic Squares into the cells(grids) of a Loub Magic Square or into the cells of its composite.

**Remark 4.7.2.** This study is based on close study of the centre piece formula C and magic sum M(S) of an Loub Magic Square relationship , an existing mighty formula of this realm.

**Theorem 4.7.3.** The centre piece C of an Loub Magic Square with first entry (first term of its underlined sequence) a and common difference j along the main column is expressed as .

**Proof.** The magic sum of the square is from High School Gaussian Formula of sum of arithmetic sequence. And, from Remark 4.7.2, .

**Theorem 4.7.4.** The centre pieces of the minimal composite Loub magic squares are expressed as where 1, 2, 3, ...**.**

**Proof.** This follows from and where C is the centre piece magic square of the composite

**Theorem 4.7.5.** The set of the centre pieces of the minimal composite Loub magic squares equipped with the integer number binary operation of addition forms an infinite additive abelian group.

**Proof.** Let C denotes the set of the centre pieces of Minimal Compositeand denotes the matrix binary operation of addition and let denotes the set of positive integer number. Then

and satisfies the **Properties of Infinite Additive Abelian Group** as follows:

1. **Closure Property.** Clearly,

is the centre piece of Composite Loub Magic Square with first term and common difference along the main column . Hence, C is closed.

1. **Associativity Property.** This is an inherited property of the set of integer number for:
2. The **Identity element** 0 is the zero face-centred magic square.
3. Given an arbitrary centre piece having first term and common difference along the main column or row of an Composite, there exists another centre piece (call the **inverse** of) of another Composite having first term and common difference along the main column or row , thus having the formulae such that
4. Obviously, **commutativity property** holds for

, we are dealing with integer number entries.

And, is an infinite additive **abelian group**.

**4.7.6. Concrete Examples of the Minimal Bull Eye Composite Loub Magic Squares Infinite Additive Abelian Groups**

**Example 1.** The following are 2 examples of the Loub Magic Square.

and

**Example 2.** The following is an example of a Composite Loub Magic Square.

**Example 3.** The following is an example of a Bull Eye Loub Magic Square of the Loub Magic Square.

We take another example of its Loub Magic Square as follows:

nd, for as:

**Remark 4.7.7.** By citing 3 examples here, we can see that we need where number of centre pieces infix adjoinment in general.

**Example 4.** The following is an example of a Minimal Bull Eye Composite Loub Magic Square.

**Theorem 4.7.8.** Composite Loub Magic Square is not isomorphic to Composite.

**Proof.**This is a proof by counter example. It is enough to provide a Composite Loub Magic Square that is not isomorphic to Composite as follows:

is not isomorphic to

**Remark 4.7.9.** See (Vasistha and Vasistha, 2006) for the definition of a group.

**Theorem 4.7.10.** The set of the Minimal Bull Eye Composite Loub Magic Square equipped with the matrix binary operation of addition forms an infinite additive abelian group.

**Proof.** Let , be arbitrarily considered, we define as follows:

1. is **closed** with respect to : From the above defination, if , as in the above.
2. is **associative**: For whence the entries of A, B, C are integer number.
3. additive **identity element** I is

, where each 0 except the first entry of the square is a series of 0s correspondingly n times.

1. Each element of has an **inverse**: If , then where –D is the square with magic sum –M(S) formed as a result of scalar multiplication of entries in D having magic sum M(S) by .

For example, the inverse of is

1. is **commutative**: whence the matrix binary operation of addition over set of integer number is commutative (inheritance).

Thus, is an infinite additive **abelian group**.

**CHAPTER FIVE**

SUMMARY, CONCLUSION AND RECOMMENDATIONS

In this chapter, we summarize and conclude our important findings as well as recommend for further researches.

5.1.**Summary**

We summarize the work in this dissertation using the consortium/ enumeration that follows:

1. The set of Loub Magic Squares form a semigroup.
2. The set of Loub Magic Squares form a group.
3. The set of Loub Magic Squares form a semiring.
4. Some aforementioned set of Loub Magic Squares forms a ring.
5. Some aforementioned set of Loub Magic Squares form a field.
6. The Transformation Semigroup of Odd Lengths is constructible using the Loub Magic Squares.
7. The Symmetry Group of Odd Lengths is constructible using the Loub Magic Squares.
8. The Tetrabonacci Group is constructible using the Loub MagicSquares.
9. There exists Rhotrix Subelement Subgroup of every Loub Magic Square.
10. There exists construction of the Loub Magic Square using matrix-rhotrix transformation.
11. Arithmetic sequence, multisets and sets are used to construct the Loub Magic Square.
12. The formulae of the centerpiece of the Loub Magic Square is introduced and proved.
13. The formulae of the magic sum of the Loub Magic Square is introduced and proved.
14. The eigen values of the Loub Magic Squares are computed and the Eigen Abelian Group is proved.
15. The De La Loub Procedure is refined.
16. Type I and type II Loub Magic Squares are defined.
17. The Loub Magic Squares are generalized.
18. The Loub Magic Squares are generalized.
19. The centre piece formula of the Composite Loub Magic Square is proved.
20. The magic sum formula of the Composite Loub Magic Square is showcased.
21. The Minimum Zero Centre Pandiagonal Type II(a) Loub Magic Square are constructed.
22. Some Loub Magic Squares are special types of some Loub Magic Squares. This is demonstrated.
23. Type I Composite of Type I Loub Magic Squares are constructed.
24. The Bull Eye Loub Magic Squares are double-handedly pioneered.
25. The Composite Bull Eye Loub Magic Square is constructed.
26. The concept of subelement is introduced.
27. Some properties of the subelements are provided.
28. Both the Cayley Theorem and the Preston-Wagner Theorem are propose to still stand. This is a proposition.
29. Room for generalizations is open.
30. The Loub Magic Square Algebraic Structure forms a new and good realm of mathematics.
31. The definition of Basic Magic Square and Magic Square (in general) is restructured.
32. The miscellany effects of rotations and/or reflections of the Type II Loub Magic Squares are extracted.
33. The set of Loub Magic Squares is partitioned via common difference.
34. The proof of square is magic when m is odd including the primes except when m is the oddest (even) prime is provided.
35. Cayley Table that is a Magic Square, the beginning of our revisionism to the reasoning of the relationship of Magic Squares and Group Theory is cited.
36. The Magic Square is isomorphic to the chosen aforementioned underlined set is proposed.
37. The magic square specialists counting the total number of Magic Square are proved wrong.
38. The relationship of the Face-Centre Magic Square and the overall Composite Magic Square is demonstrated.
39. The entries in the Loub Magic Squares can never be round/cyclic enough due to Lindemann Theorem is emphatically stressed.
40. Theorem 4.5.6.6, Remark 4.5.6.7, Theorem 4.5.6.8, Proposition 4.6.2, Theorem 4.6.4, Theorem 4.6.5, Theorem 4.6.6, Theorem 4.6.8, Theorem 4.6.9, Theorem 4.6.10, Theorem 4.7.3, Theorem 4.7.4, Theorem 4.7.5, Theorem 4.7.8 and Theorem 4.7.10 are initiated and proved.

**5.2. Conclusion**

The set of the Loub Magic Squares of the arithmetic sequence of the set of the natural number or of its multiset forms a semigroup which by analogy is refered to as the Loub Magic Squares Semigroup; and the set of the Loub Magic Squares of the arithmetic sequence of the set of integer number or of the multiset of the integer number forms a group which by analogy we refer to as the Loub Magic Squares Group.

The collection of the centre pieces with formula equipped with the binary operation of integer addition forms an abelian group and the set of all the magic sums with formula equipped with the binary operation of integer addition forms an abelian group also, where and are the corresponding first term and common difference along the main column respectively of Loub Magic Squares. It is also showcased that the set of eigen values of the set of Loub Magic Squares enclosed with the integer number operation of addition forms an abelian group. This is meaningful for the principal value of the eigen value corresponds to the magic sum, see (Daryl, 1993).

Miscellany Properties of the Loub Magic Squares are concretized via examples, see also (Babayo and Garba, 2015b). This work outlinesvia concrete examples that apt for generalizations the generalized (symbolized) and squares, the underlying sets under discussion of the squares, the rhotrix-matrix construction of the square, introduction of the terminology: Subelements of the squares, partitioning the squares, the proof of the generalized centre piece of the square as and of the magic sum M(S) as where is the first term, is the common difference along the main column expressed as , and is the last term of the arithmetic sequence, the proof of the square is magic when m is odd including the primes except when m is the oddest (even) prime, and an example of the Cayley Table Magic Square.

A basic magic square can be rotated and/or reflected to give 7 other magic squares that are considered Loubeffects. Symbolizing the Loub Squares is like moving to greater abstraction, but has generalized both the rotations and/or reflections. So far, the simplest proof is a mere intimate curious observation of the arithmetic sequence in the square. We refer you to [(Lee, 1986); Gan *et al*., 2012)]for the miscellany effects of rotations and/or reflections of the basic magic square, and we give 2 examples one for each of the generalized and squares. And, a conjecture that stimulates provision of a generalized result is presented.

Given any 3 arbitrary Loub Magic Squares such that the sum of corresponding entries of 2 chosen squares will give corresponding entries of an alike positioned cells in the 3rd one; then the set of magic sums and the set of eigen values form respective abelian groups. This result is apt for a simple generalization. Before any algebraic structure is studied/ investigated, an underlying set is firstly so considered and firstly so, we consider the multiset of natural number followed by the multiset of integer number and purposefully propose that the multiset of some rational, some real and some complex number(s)are beyond the scope of this work.

Applause is to the finest founders of one of the newest realm of mathematics, the rhotrix theory which has to do with the rhomboidal array of number; but so far, its applications are seemingly very rare or verbatim virtuoso of the matrix theory applications. A juxtaposition alike of the construction of magic squares is coiled with an infix adjoinment of zeros to be referred to the rhotrix-matrix construction for there exists an inter switch of matrix and rhotrix transformation system which forces to provide the magic squares. We provided the concrete examples of and , a provision that can be generalizednoting that square is isomorphic to one of the chosen aforementioned underlined set.

We introduce the notion of subelements and study the properties and inter properties of the magic squares subelements of the Loub Magic Squares. The most paramount important of this lies in the fact that we got a method of construction of basic magic squares that are mostly pancolumn and that have no already known method of their construction earlier. We also study the relationship of the least subelements () of the We recommend that this should also have a generalization.

We introduced the permutation and its composition over the Loub Magic Squares Semigroup logically to form a subgroup of the Symmetry Group which by analogy to the Fibonacci Group is termed the Tetrabonacci Group. The concepts of Fibonacci Semigroups (Umar, 2012) as well as the concepts of Symmetry Groups (Joseph, 2005) are widespread in the Modern Algebra Literatures. The underlying sequencewe consider herein the aforementioned magic squares is the arithmetic sequence with unity common difference. This does not sidetrack generalization, and the generalization of the general concepts revealed in this work as we finally set a conjecture. In the permutation of the Symmetry Group, only the final effect is recognized; but for the construction of the aforementioned subgroup, both the motion which may be considered as clockwise + NE-W-S or anticlockwise + NW-E-S Loub Procedures of construction where NE stands for North-East, W stands for due West, S stands for due South, and NW and E stands forthe analoguesand the effect which has to do with the final result of disjoint cycle elements of the set of permutation.

With the composition of the maps on the 4 elements per the general 8 miscellany effects of rotations and/or reflections of 1 Loub Magic Square defined on the set of their permutations , the set enclosed with its composition forms the Tetrabonacci Groupas we call for brevityan idea analogous to the Fibonacci Group. The reason for such a brevity is not too far.

Permutation over the Loub Magic Squares enclosed with its composition forms the Symmetry Group of odd lengths. Not only to generate Symmetry Groups of odd lengths with the aforementioned squares is the aim of this work, but to minimize the number of the squares that generates it. Given a Loub Magic Square, there are 7 miscellany effects ( Gan *et al.*, 2012) formed as a result of rotations and/or reflections of it if the underlying set is the set of integer number, but if it is its multiset, 4 miscellany effects are recorded for 3 are missing as a consequence of isomorphism.

If a permutation is defined on the collection of the squares and their effects, the collection of the maps and its composition forms the Symmetry Group of odd length. The odd length is because the permutations are over the Loub Magic Squares and Loub Magic Squares are of the form where n is odd except , the triviality.It will be remarkable if the number of Loub Magic Squares is minimized. This is also an aim of this work. To economize space, we showcased that this is true for Symmetry Group of length 3 and no deep reasoning is required to make such a sudden conclusion of generality.

The job here, however along with caution with the Cayley Theorem, is to find the minimum number of the Loub Squares that will completely cover the structure of all the elements of the Transformation Semigroup of odd lengths partial map. If the Loub Magic Square subsets are able to construct both the permutation subsets of the collection of maps from a set to itself and both the maps form respective SymmetryGroup (Vasistha and Vasistha, 2006) and (Joseph, 2005) and Transformation Semigroup (Howie, 2003) with respect to the composition of maps and the set of the Loub Magic Squares equipped or enclosed with matrix binary operation of addition is an abelian group, then the Cayley Theorem for both the group and the semigroup still stand while all the job joyously jugs for the subsets of the squares so considered are not groups or semigroups by themselves.

We know it exactly may be possible we can define elementary row and column operations on a Loub Magic Squares to construct the Transformation Semigroup, but never attempted it for the higher the number of entries in the set n of the Transformation Semigroup , the more cumbersome is such a set of all such maps that are apt to be equipped with composition to form it. The order of is and the order of , , is , and In this work, it is showcased that the number of the Loub Magic Square mappings will generate number of all maps to be equipped with composition to form the Transformation Semigroup. To economize space, we present a concrete example of the since Loub Magic Squares is a triviality and does not exist. The n has to always be odd matching the exact definition of the Loub Magic Square.

The Rhotrix Subelements of the square are the rhomboidal array of number that are sitting in the magic squares. The matrix subelements have analogous meaning and are considered babyish tautology, but we need the 2 arrays together to get the method: Rhotrix-Matrix Construction of the Loub Magic Squares. This implies that one of the newest realm of mathematics, the Rhotrix Theory has now gotten another genuine application. We showcased via concrete examples that the Rhotrix Subelements of the aforementioned squares form infinite additive abelian group. The aim here are two: First, to present the Rhotrix-Matrix Construction of and Magic Squares where the is Loub The construction of Magic Squares via Rhotrix-Matrix Method is one of the finest achievement of one of the newest realm of mathematics, the Rhotrix Theory first introduced by (Ajibade, 2003). Secondly, we explicate that the Rhotrix Subelements of the Loub Magic Squares form an infinite additive abelian group.

This work is pioneering investigation of the Loub Magic Squares over multiset of rational number of the form infinite algebraic field, where denotes the multiset of integer number. By the Loub Magic Squares L (as we denote it), we understand the set of magic squares formed by the De La Loub Procedure. It is explicated that the set equipped with the binary operation of matrix addition (as we denote it) forms an infinite additive abelian group, and the set enclosed with the rational number multiplication (as we denote it) forms an infinite multiplicative abelian group if the underlying set so considered of the entries of the aforementioned squares is the multiset of the aforementioned set of number. forms an infinite field. The Loub Magic Squares over the multiset of rational number of the form forms an infinite multiplicative abelian group,, thus, making an infinite field, see also (Babayo and Garba, 2015a).

Arbitrarily, Semi Pandiagonal Loub Magic Squares are considered not squares such that to economize space, and not such a square to dodge near bias choice. The multiset of rational number of the form is aptly considered even though the multiset of rational number of the form, a special type of scalar multiplication, will do. Also, we consider the sequence c than a, b, c, ... n times, a, b, c, ... n times, ... n times though the later will also give an analogous result; but presenting both the two is also a babyish tautology.

In this work, mystic miscellaneous algebraic properties of the set of Composite (Nested) Loub Magic Squares are vividly visualized [see also (Babayo and Garb, 2015j)]. And, verbatim virtuoso of algebraic properties of the Loub Magic Squares viz: Eigen Group, Magic Sum Group and Centre Pieces Group elegantly viewed the algebraic properties of its Composite. It is also showcased that both the 2 sets equipped with the binary operation of matrix addition form infinite additive abelian groups.It is remarkable that almost joyously the sets of eigen values, centre pieces and magic sums of the Loub Magic Squares Infinite Abelian Group form Infinite Additive Abelian Groups. For magic squares eigen values computations, see (Daryl, 1993).

We highlighted consortium of miscellany effects of rotations and/or reflections [see also Gan *et al.,*2012)] and/or enumerations of the Loub Magic Squares to figure out the consortium of the Composites. Establishing such a fact relationship set us conjecture that the Composite (Ahmed, 2004) Loub Magic Squares Infinite Additive Abelian Group is a miscellany case of the Loub Magic Squares Infinite Abelian Group.

By Type I Composite of Type I Loub Magic Squares, we understand the set of all Type I Loub Magic Squares such that each cell (grid) of its element is a Type I Loub Magic Square, the magic squares formed by the De La Loub Procedure over the arithmetic sequence of common difference of 1.

It is explicated that the aforementioned set equipped with the matrix binary operation of addition(as we denote it) forms an infinite additive abelian group as well as we showcased that the set of Type I Composite of Type II Loub Magic Squares, the magic squares constructed with the Repeating-Pattern-Sequence Loub Procedure, equipped with the aforementioned operation and with the rational number multiplication (as we denote it) forms an Infinite Field if the underlying multiset of entries of the squares is of the rational number of the form. forms an infinite field.

The set of Loub Magic Squares of type I equipped with the binary operation of matrix addition forms an infinite additive abelian group if the underlying set is of the multiset of integer number, and the set of Loub Magic Squares of type II equipped with the binary operation of matrix addition and with the rational number multiplication forms an infinite field if the underlying multiset is of integer number including some rational number of the form. Also, we delved into the Minimum Zero-Centre Pandiagonal Composite Type II(a) Loub Magic Squares over Multiset of Natural Number Semiring [see also (Babayo *et al*., 2015)].

It is remarkable if the composite of the aforementioned types of magic squares form the aforementioned respective groups and fields. This is our construction, and however along with the scope, a simple definition, a finer procedure and a newest generation of the Loubmanifested. We also established a fascinating relationship between the face- centre-magic squares and the overall composite magic sums, a relationship so close in look and in application in this realm as the Einstein Equation .

Concrete examples of the Minimal Bull Eye Composite Loub Magic Square Infinite Additive Abelian Groups are presented. The minimum Loub Magic Square is the and so the minimal Composite Loub Magic Square is the . We construct its Bull Eye and proved that the Bull Eyealso forms an Infinite Additive Abelian Group. Here, Bull Eye Loub Magic Squares are constructed as well as Bull Eye Composite.We showcased that the set of the Minimal Bull Eye Compositeforms an infinite additive abelian group, an idea that does not require a deep mathematics to conclude its generalization.

The whole concept is based on intimate survey of the structure of the centre piece and magic sum relationship formula, the mighty formula of magic squares realm. We will rightly so introduce a new result on it.The minimum Loub Magic Square is the for Square is trivial for it is isomorphic to the underlined set of entries of the square so considered and Loub does not exist. Since the minimum is considered to be the , the minimum compositeis and its minimal Bull Eye is because an infix adjoinment of 17 times the face-centre magic squares is more than necessary (Babayo and Garba, 2015*l*).

**5.3.Recommendations**

We recommend that further research be carried out on the following titles: The Tetrabonacci Subgroup of the Symmetry Group over the Loub Magic Squares Semigroup, Rhotrix Subelement Subgroups of the Loub Magic Squares Infinite Additive Abelian Group, Symmetry Group of Odd Lengths over the Loub Magic Squares Semigroup of Multiset of Natural Number, Transformation Semigroup of Odd Lengths over the Loub Magic Squares Semigroup of Multiset of Natural Number, Loub Magic Square Nearings, Loub Magic Squares over Multiset of Rational Number Infinite Field, Composite Loub Magic Squares Infinite Abelian Group Miscellany Case of the Loub Magic Squares Infinite Abelian Group, Type I Composite of Type I Loub Magic Squares Infinite Additive Abelian Group and of Type II Infinite Field, and Concrete Examples of the Minimal Bull Eye Composite Loub Magic Square Infinite Additive Abelian Groups. We recommend that the above titles will be constructed over another known magic squares viz: Franklin Squares, Doubly Even Magic Squares, Even Ordered Magic Squares, Lo Shu Magic Squares in General, General Magic Squares, etc.

We also recommend that further research be carried out on the following: Pure algebraic development of Loub Magic Squares, the Green’s Relations of the Loub Magic Squares Semigroup, the Green’s Relations of the Rhotrix Subelement Subsemigroups of the Loub Magic Squares Semigroup, the Ring of Integer’s Action on the Loub Magic Squares Set and its Module or its Vector Space, Rees’ Magic Squares Semigroup, the Generalized Loub Procedure, the Generalized Rhotrix-Matrix Construction of the Loub Magic Squares, Factor Group of the Matrix Group of the Normal Loub Magic Square Subgroups and lots more.

We finally recommend theoretical development of the following titles: The Tribonacci Subgroup of the Symmetry Group over the Loub Magic Squares Semigroup, Rhotrix Subelement Subgroups of the Composite Loub Magic Squares Infinite Additive Abelian Group, Rhotrix-Matrix Construction of the Loub Magic Squares, Rhotrix Subelement Subgroups of the Pandiagonal and Bull Eye Loub Magic Squares Infinite Additive Abelian Group,Symmetry Group of Even Lengths over Magic Squares Semigroup of Multiset of Natural Number, Transformation Semigroup of Even Lengths over Magic Squares Semigroup of Multiset of Natural Number, Loub Magic Squares over Multiset of Rational Number Infinite Field, Composite Magic Squares Infinite Abelian Group Miscellany Case of the Loub Magic Squares Infinite Abelian Group, Type II Composite of Type I Loub Magic Squares Infinite Additive Abelian Group,Type II Composite of Type II Loub Magic Squares Infinite Field, Concrete Theorems of the Minimal and Non-Minimal Bull Eye Composite Loub Magic Square Infinite Additive Abelian Groups, Applications of Cosets, Conjugates, Orbits Alike Operations as a Miscellany Case of Common Difference Partition of Loub Magic Squares Finite Abelian Groups, Semifields over the Loub Magic Squares,Set of Loub Magic Squares Finite Abelian Groups, Inclusive and Exclusive Permutations over Loub Magic Squares, Construction of Other Magic Squares through Others viz: Construction of Bull Eye Composite of Even Ordered Magic Squares, Dihedral Group over Magic Squares Semigroup of Multiset of Natural Number,Subelement Subsemigroups of the Loub Magic Squares Inverse Semigroups over Multiset of Natural Number Including Zero, Solutions of Systems of Differential Equations Having Magic Square Arrays of Coefficients, Applications of Loub Magic Squares to Astronomy and one day, but only one day, we will proof that every finite algebraic structure is embedded in the Loub Magic Square Algebraic Structures, the Cayley Theorem of this realm.Our research is the door to all the above researches. A search is not a research if it is not a door to many researches, we never forget that.

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