Continental J. Applied Sciences 8 (2): 34 - 39, 2013
ISSN: 1597 – 9928
http://www.wiloludjournal.com
doi:10.5707/cjapplsci.2013.8.2.34.39
Printed in Nigeria

EFFICIENCY COMPARISON OF RATIO TYPE ESTIMATORS

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ABSTRACT
The efficiencies of the ratio type estimators have been increased by using linear transformation on auxiliary variable in the literature. Keeping in view, a ratio type estimator of the population mean of variable of interest has been proposed involving unknown weights. Theoretically, its bias and mean square error (mse) have been obtained and compared with other conventional and considered estimators. By this comparison, the conditions that make the proposed estimator more efficient than others have been found. Optimum bias and mean square error of proposed estimator have also been obtained. Six natural populations have been used for empirical comparisons and the estimator that is more efficient and preferred has been obtained.

KEYWORDS: auxiliary variable, bias, efficiency, mean square error, optimum.

Received for Publication: 06/02/13 Accepted for Publication: 28/04/13
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INTRODUCTION

Cochran made a particularly important contribution to the modern sampling theory by suggesting methods of utilizing the auxiliary information for the purpose of estimation of the population mean $\bar{Y}$ in order to increase the precision of the estimates. Several well known procedures use auxiliary information at the estimation stage. This is the most commonly used-way of utilizing auxiliary information, which gives rise to some estimators that are known today, and in under certain conditions, these estimators are more efficient than the sample, mean $\bar{y}$ estimator. Several researchers diverted their attention to modifying estimation procedures, so that unbiased or less biased estimators could be obtained. The work of Hartley and Ross (1954) deserved special attention in this direction. Several other authors also proposed unbiased or almost unbiased ratio type estimators. Tin (1965) proposed bias adjusted ratio estimators, which are equally efficient in finite population. Rao (1981b) also suggested some almost unbiased ratio type estimators.

Consequently, a number of modified ratio estimators came into existence in recent past, when such estimators are generally developed either using one or more unknown constants or introducing a convex linear combination of sample and population means of auxiliary characteristic with unknown weights.In both cases, optimum choices of unknown parameters are made by minimizing the mean square error of the modified estimators, so that they become superior than the conventional one, Singh and Tailor (2003), Singh, Singh and Singh (1991). Also, Srivastava (1967), Kadilar and Cingi (2004) and others have suggested modified ratio type estimators and studied their properties, theoretically and empirically.

In literature, it has been shown by various authors such as Mohanty and Das (1971), Reddy (1974), Srivenkataramana and Srinath (1976), Srivenkataramana (1978), Chaudhuri and Adikari (1979), that the bias and mse of the ratio estimator $\bar{y}_R$ can be reduced with the application of transformation on the auxiliary variable x. The use of auxiliary information can increase the precision of an estimator when study variable y is highly correlated with auxiliary variable x. for example, if x_i is the value of y_i at some previous exercises, the ratio method, uses the sample to estimate the relative change Y ⁄ X that has occurred .The estimated relative change y ⁄ x is multiplied by the known population total X on the previous occasion to provide an estimate of the current population total. If the ratio y_i ⁄ x_i is nearly the same in all sampling units, the values y ⁄ x vary little from one sample to another, and the ratio estimation is of high precision. Relating this to the data to be used in this dissertation, x_i is the results of a course Descriptive Statistics I, which is a pre-requisite to another course y_i Descriptive Statistics II. Example among others include: amount of milk produced by cows y and a particular breed of cows x; amount of yield of wheat crop y and a particular variety of wheat x; the results of students that wrote mathematics examination in West African Examination Council (WAEC) y and the total number of students that sat for WAEC examinations in that year of interest x, and so on.

Adewara (2006) compared empirically the following estimators:

(i) simple mean estimator $\bar{y}$ (ii) $\hat{R}_1 = \frac{\bar{y}}{\bar{x}}$ (iii) $\hat{R}_2 = \frac{\bar{y}^*}{\bar{x}^*}$ (iv) $\hat{R}_3 = \frac{\bar{y}^*}{\bar{x}}$ (v) $\hat{R}_4 = \frac{\bar{y}}{\bar{x}^*}$

In the present paper, we have proposed a ratio type estimator of population mean $\bar{Y}$.The expression for its bias and mean square error has been derived. The theoretical comparisons of the proposed estimator with the existing ones have been made with respect to their mean square errors.

Notations:

Let Y be the characteristic under study and Y_i be the value of the characteristic associated with the ith unit in the population i=1,2,…,N. X be the auxiliary characteristic and X_i denote the value of auxiliary characteristic for the ith unit in the population.

$\bar{X}$ and $\bar{Y}$ are the population means of variable of the auxiliary variable X and the variable of interest Y respectively.

$R = \frac{\bar{Y}}{\bar{X}}$ denotes the ratio of the population means.

$\widehat{R}_n = \frac{\bar{y}}{\bar{x}}$ denotes the ratio of the sample means.

$\bar{x}^*$ denotes the mean of the auxiliary variable x yet to be drawn.

$\bar{y}^*$ denotes the mean of the variable of interest y yet to be drawn.

C_x^2 denotes the square of the coefficient of variation of the auxiliary variable x.

C_y^2 denotes the square of the coefficient of variation of the variable of interest y.

ρ_{xy} denotes the correlation coefficient between x and y.

Proposed Estimator:

The proposed estimator is given by

$$\hat{R}_{pr} = \frac{\bar{y}}{\alpha\bar{x}^* + (1-\alpha)\bar{X}}$$

The bias and the MSE of the proposed estimator have been obtained under the two cases in which the relationship among $\bar{X}$, $\bar{x}$ and $\bar{x}^*$ Adewara (2006) are as follows:

Case 1: $\bar{X} = \theta\ \bar{x} + (1-\theta)\ \bar{x}^*$ where $\bar{x}^* = \bar{X}[1 - \varphi\ \Delta_{\bar{x}}]$

Similarly $\bar{Y} = \theta\ \bar{y} + (1-\theta)\bar{y}^*$ where $\bar{y}^* = \bar{Y}[1 - \varphi\ \Delta_{\bar{y}}]$, where $\theta = {}^1/_n - {}^1/_N$

Case 2: $\bar{X} = f\ \bar{x} + (1-f)\ \bar{x}^*$ where $\bar{x}^* = \bar{X}[1 - \psi\ \Delta_{\bar{x}}]$

Similarly $\bar{Y} = f\ \bar{y} + (1-f)\ \bar{y}^*$ where $\bar{y}^* = \bar{Y}[1 - \psi\ \Delta_{\bar{y}}]$, where $f = {}^n/_N$

Bias and MSE of Proposed Estimator

Using Taylor's expansion under usual assumptions, the bias and mean square error of proposed estimator to 0 (n^{-1}) are obtained as follows :

Case I : Bias$(\hat{R}_{pr})$ = $\left(\frac{1}{n}-\frac{1}{N}\right)\ \varphi\,\alpha\ (\rho C_x C_y+\ \varphi\,\alpha C_x^2)$

Mse $(\hat{R}_{pr})$ $= R^2\ \left(\frac{1}{n}-\frac{1}{N}\right)(C_y^2+2\varphi\,\alpha\rho C_x C_y+\ \varphi^2\alpha^2 {C_x}^2)$, Where $\varphi\ =\ (\frac{N-n}{Nn-N+n})$

Case II: Bias$(\hat{R}_{pr})$ = $\left(\frac{1}{n}-\frac{1}{N}\right)\psi\alpha\ (\rho C_x C_y+\psi\alpha C_x^2)$

Mse $(\hat{R}_{pr})$= $R^2\ \left(\frac{1}{n}-\frac{1}{N}\right)(C_y^2+2\psi\alpha\rho C_x C_y+\psi^2\alpha^2 {C_x}^2)$ where $\psi\ =\ \frac{n}{N-n}$

Bias and MSE of Considered Estimators:

Case I: Bias $(\bar{y})$ = 0

Bias $(\hat{R}_1)$ = R$\left(\frac{N-n}{Nn}\right)(C_x^2-\rho C_x C_y)$, R = $\frac{\bar{Y}}{\bar{X}}$

Bias $(\hat{R}_2)$ = − R$\left(\frac{N-n}{Nn}\right)(\frac{N-n}{Nn-N+n})\ (C_x^2+\rho C_x C_y)$

Bias $(\hat{R}_3)$ = R$\left(\frac{N-n}{Nn}\right)\left(C_x^2+(\frac{N-n}{Nn-N+n})\rho C_x C_y\right)$

Bias $(\hat{R}_4)$ = R$\left(\frac{N-n}{Nn}\right)\left((\frac{N-n}{Nn-N+n})\rho C_x C_y-(\frac{N-n}{Nn-N+n})^2 C_x^2\right)$

Mse $(\bar{y})$ = $\bar{Y}^2\ \left(\frac{N-n}{Nn}\right)C_y^2$

Mse $(\hat{R}_1)$ = $R^2\left(\frac{N-n}{Nn}\right)(C_y^2-2\rho C_x C_y+C_x^2)$

Mse $(\hat{R}_2)$ = $R^2\ \left(\frac{N-n}{Nn}\right)(\frac{N-n}{Nn-N+n})^2(C_y^2-2\rho C_x C_y+C_x^2)$

Mse $(\hat{R}_3)$ = $R^2\left(\frac{N-n}{Nn}\right)\left((\frac{N-n}{Nn-N+n})^2 C_y^2-2(\frac{N-n}{Nn-N+n})\rho C_x C_y+C_x^2\right)$

Mse $(\hat{R}_4)$ = $R^2\left(\frac{N-n}{Nn}\right)\left(C_y^2-2(\frac{N-n}{Nn-N+n})\rho C_x C_y+(\frac{N-n}{Nn-N+n})^2 C_x^2\right)$

Case II : Bias $(\bar{y})$ = 0

Bias $(\hat{R}_1)$ = R$\left(\frac{N-n}{Nn}\right)(C_x^2-\rho C_x C_y)$, R = $\frac{\bar{Y}}{\bar{X}}$

Bias $(\hat{R}_2)$ = − R$\left(\frac{N-n}{Nn}\right)(\frac{n}{N-n})\ (C_x^2+\rho C_x C_y)$

Bias $(\hat{R}_3)$ = R$\left(\frac{N-n}{Nn}\right)\left(C_x^2+(\frac{n}{N-n})\rho C_x C_y\right)$

Bias $(\hat{R}_4)$ = R$\left(\frac{N-n}{Nn}\right)\left((\frac{n}{N-n})\rho C_x C_y-(\frac{n}{N-n})^2 C_x^2\right)$

Mse$(\bar{y})$ = $\bar{Y}^2\ \left(\frac{N-n}{Nn}\right)C_y^2$

Mse$(\hat{R}_1)$ = $R^2\left(\frac{N-n}{Nn}\right)(C_y^2-2\rho C_x C_y+C_x^2)$

Mse $(\hat{R}_2)$ = $R^2\left(\frac{N-n}{Nn}\right)(\frac{n}{N-n})^2(C_y^2-2\rho C_x C_y+C_x^2)$

Mse $(\hat{R}_3)$ = $R^2\left(\frac{N-n}{Nn}\right)\left((\frac{n}{N-n})^2 C_y^2-2(\frac{n}{N-n})\rho C_x C_y+C_x^2\right)$

Mse $(\hat{R}_4)$ = $R^2\left(\frac{N-n}{Nn}\right)\left(C_y^2-2(\frac{n}{N-n})\rho C_x C_y+(\frac{n}{N-n})^2 C_x^2\right)$

Comparison of the Mean Squares Errors (MSE)

It is appropriate to explore the situations under which the proposed estimator is more efficient than the considered estimators. We present here the comparisons of $\hat{R}_{pr}$ with $\hat{R}_1$, $\hat{R}_2$,$\hat{R}_3$ and $\hat{R}_4$ under both cases :

CaseI:

(a) Comparing the mean square error of $\hat{R}_{pr}$ with $\hat{R}_1$, we observed that $\hat{R}_{pr}$ is preferable over $\hat{R}_1$ if $\alpha \in \left(\frac{1}{\varphi}, \frac{2\rho C_y - C_x}{\varphi\, C_x}\right)$

(b) Comparing the mean square error of $\hat{R}_{pr}$ with $\hat{R}_2$, we observed that $\hat{R}_{pr}$ is preferable over $\hat{R}_2$ if $\alpha \in \left(\frac{\sqrt{(\rho^2 C_y^2 - \delta)} - \rho C_y}{\varphi\, C_x}, \frac{-\rho C_y - \sqrt{(\rho^2 C_y^2 - \delta)}}{\varphi\, C_x}\right)$

Where $\delta = (1 - \varphi^2)\left(C_y^2 + 2\varphi^2 \rho C_x C_y - \varphi^2 {C_x}^2\right)$

(c) $\hat{R}_{pr}$ is preferable over $\hat{R}_3$, if

$$\alpha \in \left(\frac{\sqrt{(\rho^2 C_y^2 - d)} - \rho C_y}{\varphi\, C_x}, \frac{-\rho C_y - \sqrt{(\rho^2 C_y^2 - \delta)}}{\varphi\, C_x}\right)$$

and d = $(1 - \varphi^2)\left(C_y^2 + 2\varphi\, \rho C_x C_y - {C_x}^2\right)$.

(d) Again $\hat{R}_{pr}$ is preferable over $\hat{R}_4$, if $\alpha \in \left(-1, 1 - \frac{2\rho C_y}{\varphi\, C_x}\right)$

Case II:

(a) Comparing the mean square error of $\hat{R}_{pr}$ with $\hat{R}_1$, we observed that $\hat{R}_{pr}$ is preferable over $\hat{R}_1$ if $\alpha \in \left(\frac{1}{\psi}, \frac{2\rho C_y - C_x}{\psi\, C_x}\right)$

(b) Comparing the mean square error of $\hat{R}_{pr}$ with $\hat{R}_2$, we observed that $\hat{R}_{pr}$ is preferable over $\hat{R}_2$if $\alpha \in \left(\frac{\sqrt{(\rho^2 C_y^2 - \delta)} - \rho C_y}{\psi\, C_x}, \frac{-\rho C_y - \sqrt{(\rho^2 C_y^2 - \delta)}}{\psi\, C_x}\right)$

Where $\delta = (1 - \psi^2)\left(C_y^2 + 2\psi^2 \rho C_x C_y - \psi^2 {C_x}^2\right)$

(c) $\hat{R}_{pr}$ is preferable over $\hat{R}_3$, if $\alpha \in \left(\frac{\sqrt{(\rho^2 C_y^2 - d)} - \rho C_y}{\psi\, C_x}, \frac{-\rho C_y - \sqrt{(\rho^2 C_y^2 - \delta)}}{\psi\, C_x}\right)$

and d = $(1 - \psi^2)\left(C_y^2 + 2\psi\, \rho C_x C_y - {C_x}^2\right)$.

(d) Again $\hat{R}_{pr}$ is preferable over $\hat{R}_4$, if $\alpha \in \left(-1, 1 - \frac{2\rho C_y}{\psi\, C_x}\right)$

Optimum Bias and MSE of proposed Estimator:
The optimum value of constant α, for which the value of bias and mean square error should be minimum, have been obtained.

For bias, the optimum value of α is obtained as

$$\alpha = \frac{-\rho C_y}{2\theta C_x}$$

and optimum bias of the proposed estimator is

Bias $(\hat{R}_{pr})_{\text{opt}} = \frac{R}{4}\left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 \rho^2$

Again for mean square error,the optimum value of α is given by

$$\alpha = \frac{-\rho C_y}{\theta C_x}$$

and the optimum mean square error of proposed estimator is

MSE $(\hat{R}_{pr})_{opt} = R^2 \left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 (1 - \rho^2)$

This is equivalent to mean square error of regression estimator.

Empirical Comparison:

To illustrate the working of the proposed estimator six populations have been taken. These six populations are unpublished data, collected from the Department of Mathematics and Statistics, Niger State Polytechnic, Zungeru, Niger State, 2004/2005-2009/2010 academic sessions .Y represent the scores in Descriptive Statistics II and X represent the corresponding scores in Descriptive Statistics I. The absolute biases and relative efficiencies of $\bar{y}$, $\hat{R}_{pr}$, $\hat{R}_1$, $\hat{R}_2$,$\hat{R}_3$ and $\hat{R}_4$ have been presented in Table 1(for case I)and Table 2 (for case II). The results in Table1 and Table 2 reveal that the proposed estimator is least bias and more efficient than other existing estimators except $\hat{R}_2$.

Table 1 (Case I) : Bias and Relative Efficiencies of $\bar{y}$, $\hat{R}_{pr}$ $\hat{R}_1$, $\hat{R}_2$,$\hat{R}_3$ and $\hat{R}_4$:

	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
Bias ($\bar{y}$)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Bias ($\hat{R}_{pr}$)	3.3×10^{-7}	-1.1×10^{-7}	9.8×10^{-7}	-5.6×10^{-9}	2.56×10^{-7}	-3.8×10^{-7}
Bias ($\hat{R}_1$)	4.8×10^{-3}	1.6×10^{-3}	7.2×10^{-4}	7.2×10^{-4}	1.6×10^{-4}	1.1×10^{-3}
Bias ($\hat{R}_2$)	-2.3×10^{-4}	-1.0×10^{-4}	-1.1×10^{-4}	-2.9×10^{-5}	-8.0×10^{-6}	-1.2×10^{-4}
Bias ($\hat{R}_3$)	6.1×10^{-3}	2.9×10^{-3}	3.1×10^{-3}	1.4×10^{-3}	3.3×10^{-4}	3.2×10^{-3}
Bias ($\hat{R}_4$)	3.2×10^{-5}	2.9×10^{-5}	4.5×10^{-5}	1.3×10^{-5}	4.8×10^{-6}	4.5×10^{-5}
R.E. ($\bar{y}$)	100.00	100.00	100.00	100.00	100.00	100.00
R.E. ($\hat{R}_{pr}$)	1.6×10^{5}	2.9×10^{5}	4.7×10^{5}	3.5×10^{5}	4.4×10^{5}	3.5×10^{5}
R.E. ($\hat{R}_1$)	4.1×10^{4}	1.7×10^{5}	4.2×10^{5}	3.3×10^{5}	4.1×10^{5}	2.7×10^{5}
R.E. ($\hat{R}_2$)	4.0×10^{7}	2.9×10^{8}	1.1×10^{9}	5.8×10^{10}	4.2×10^{9}	4.9×10^{8}
R.E. ($\hat{R}_3$)	3.6×10^{4}	1.4×10^{5}	2.2×10^{5}	2.8×10^{5}	1.5×10^{9}	1.4×10^{5}
R.E. ($\hat{R}_4$)	1.4×10^{5}	2.1×10^{5}	2.1×10^{5}	2.0×10^{5}	2.7×10^{5}	1.2×10^{5}

Table 2 (Case II): Bias and Relative Efficiencies of $\bar{y}$, $\hat{R}_{pr}$, $\hat{R}_1$, $\hat{R}_2$,$\hat{R}_3$, and $\hat{R}_4$:

	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
Bias ($\bar{y}$)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Bias ($\hat{R}_{pr}$)	2.4×10^{-8}	1.3×10^{-7}	-6.3×10^{-7}	5.9×10^{-8}	6.5×10^{-5}	-6.8×10^{-7}
Bias ($\hat{R}_1$)	4.8×10^{-3}	1.6×10^{-3}	7.2×10^{-4}	7.2×10^{-4}	1.6×10^{-4}	1.1×10^{-3}
Bias ($\hat{R}_2$)	-4.4×10^{-3}	-2.8×10^{-3}	-4.0×10^{-3}	-1.7×10^{-3}	-4.5×10^{-4}	-3.9×10^{-3}
Bias ($\hat{R}_3$)	6.7×10^{-3}	3.7×10^{-3}	4.8×10^{-3}	2.1×10^{-3}	6.0×10^{-4}	4.6×10^{-3}
Bias ($\hat{R}_4$)	1.6×10^{-4}	-4.7×10^{-4}	3.2×10^{-5}	8.0×10^{-5}	1.7×10^{-4}	2.9×10^{-4}
R.E. ($\bar{y}$)	100.00	100.00	100.00	100.00	100.00	100.00
R.E. ($\hat{R}_{pr}$)	1.5×10^{5}	3.9×10^{5}	6.3×10^{5}	4.8×10^{5}	6.3×10^{5}	4.5×10^{5}
R.E. ($\hat{R}_1$)	4.1×10^{4}	1.7×10^{5}	4.2×10^{5}	3.3×10^{5}	4.1×10^{5}	2.7×10^{5}
R.E. ($\hat{R}_2$)	1.1×10^{5}	3.8×10^{5}	7.5×10^{5}	6.5×10^{5}	1.6×10^{6}	4.6×10^{5}
R.E. ($\hat{R}_3$)	4.2×10^{4}	1.9×10^{5}	4.9×10^{5}	4.6×10^{5}	1.4×10^{6}	2.8×10^{5}
R.E. ($\hat{R}_4$)	9.1×10^{4}	2.6×10^{5}	4.7×10^{5}	3.5×10^{5}	3.5×10^{5}	3.4×10^{5}

CONCLUSIONS

$\hat{R}_{pr}$ performed outstandingly well against the biases of the considered estimators $\hat{R}_1, \hat{R}_2$, $\hat{R}_3$ and $\hat{R}_4$ in both the cases. In addition, Mse of $\hat{R}_{pr}$ is better in larger proportion of the results, shown in the empirical analysis, except $\hat{R}_2$.

Theoretical comparison of the proposed estimator is made with the considered estimators and the conditions at which the proposed estimator is better than the considered estimators are stated.

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